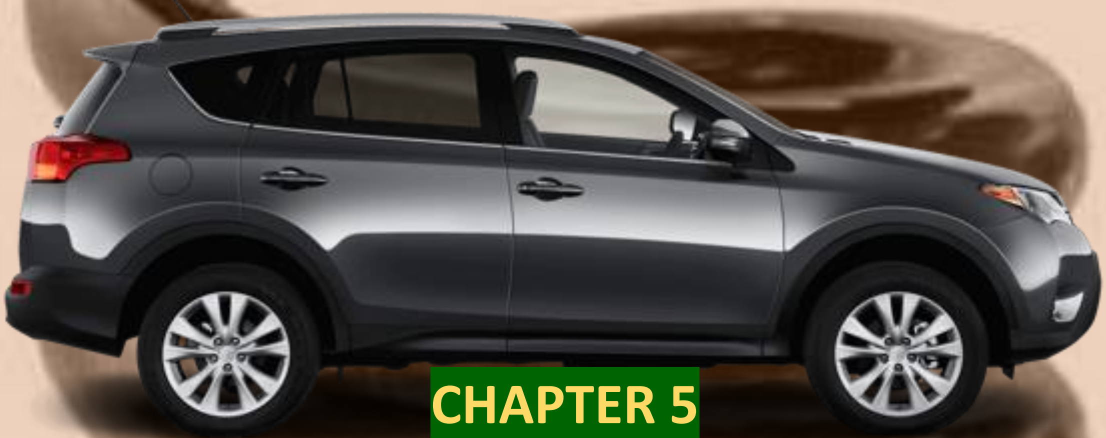
PROJECT 2
FATIGUE ANALYSIS OF HELICAL COIL COMPRESSION SPRING FROM TOYOTA RAV4 XLE 2015



CHAPTER 5
Fatigue Tests/Models, Failure Criteria and Fatigue Failure Mechanisms

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Table of Content

Chapter 5: Fatigue Tests/Models, Failure Criteria and Fatigue Failure Mechanisms

In the previous chapters, we have built a significant amount of theory to get started on fatigue analysis. As mentioned previously, we are analysing the helical coil compression spring and are interested in calculating the life expectancy of this spring based on a detailed fatigue analysis. Till now, we have represented the numerous stresses that act on the spring in the form of shear stresses. We have represented this in two kinds of graphs – minimum – maximum and alternate – mean stress diagrams. The goal of this chapter is to facilitate this movement from these graphs into carrying out a fatigue analysis. In this chapter, we will introduce fatigue models in order to prepare for coming up with a life expectancy for the spring.

Section 5.1: Fatigue Factors

Before we get started with the fatigue analysis, there are some fatigue factors we need to have in hand as tools to prepare ourselves for fatigue analysis in the next chapter. To plot the very basic stress-strain curve, the specimen is pulled from both the directions. A strain gage measures the deformation through which the strain can be calculated. From the force used to pull the specimen, a normal stress can be calculated. The figure 5.1-1 below shows this stress strain curve.[i]

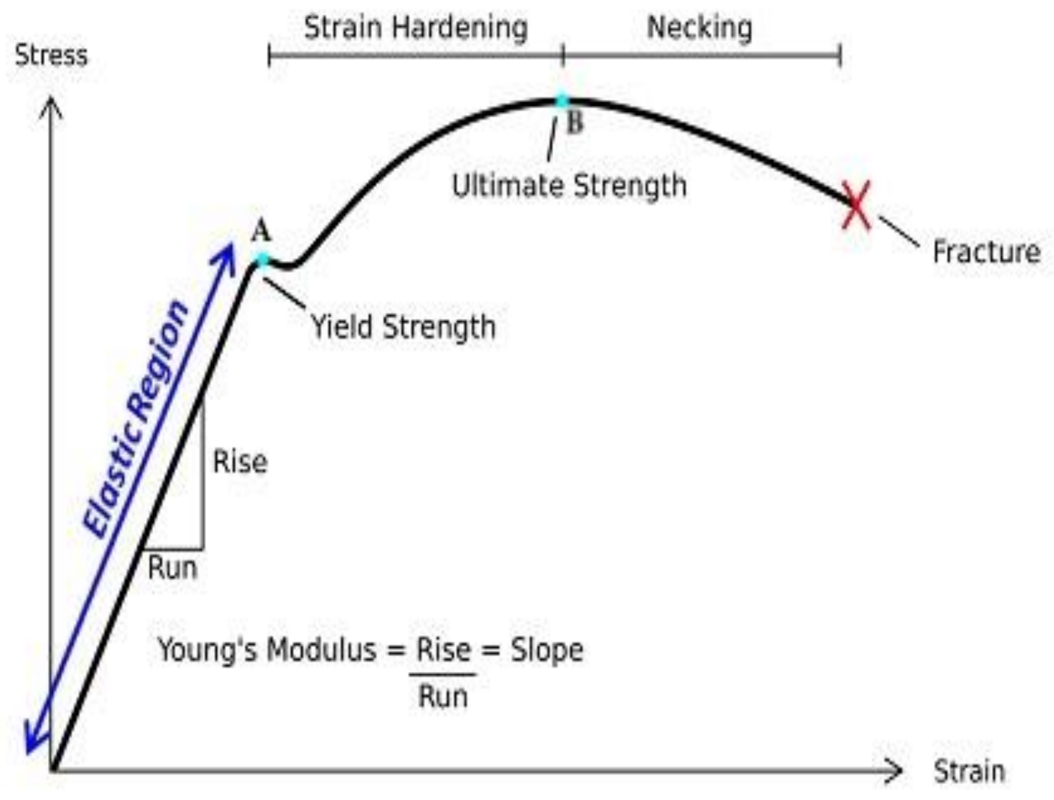

Figure 5.1-1: Stress and strain curve for an unidentified specimen

The point A and B in the above figure specify the yield and the tensile stress. When a load is applied, the specimen continues to deform at a linear rate. The stress at which this linear rate stops at point A is called the yield strength. In carrying out this experiment, the maximum stress that the specimen experiences at point B is called the ultimate tensile strength. These values for the yield and the tensile stress have been calculated and listed in the table 5.1-1 below.

Table 5.1-1: Ultimate tensile and yield stress of the spring

Notation	Definition	Metric value	Units	English value	Units
(S_u)	Ultimate Tensile Stress	1600	MPa	232.06	ksi
(S_y)	Tensile Yield Stress	1440	MPa	208.85	ksi

The above table is for the when only normal forces are acting. However, we discovered in chapter 3 that there are no normal forces acting on the spring. There are only shear forces acting. Therefore, we need to modify the above values to make sure they fit the appropriate shear stress models.

When the above test is done and the specimen fails, we would expect the fracture to happen in a single plane. However, surprisingly, it happens at a 45° and this is due to a smaller shear force acting, which is a fraction of the total yield and tensile strength. This has been proved in theory again and again and is shown in the figure 5.1-2 below and is the reason why this steel experiences a failure at 45°.

Figure 5.1-2: Biaxial testing of steel and 45°

This is why we have extracted the shear yield and ultimate stresses, which are more apt for our fatigue analysis below.[ii] As mentioned earlier, they are fractions of the original yield and tensile stresses.

$$S_{us} = 0.8\ S_u \qquad \text{Equation 5.1-1}$$

$$S_{ys} = 0.577\ S_y \qquad \text{Equation 5.1-2}$$

These values have been calculated and placed in the table 5.1-2 below.

Table 5.1-2: Shear yield and tensile stress of the spring

Notation	Definition	Metric value	Units	English value	Units
(S_{us})	Ultimate Shear Stress	1280	MPa	185.65	Ksi
(S_{ys})	Shear Yield Stress	831	MPa	120.58	ksi

The helical coil compression spring is produced using a process called shot pinning as it increases the spring performance significantly. However, this process is not completely understood. In this process, the whole surface of the spring is bombarded with many small particle of rounded shots. These are hit with such high velocities that about 80% of the spring is covered with small indents. In fact, it is required by ASTM that this minimum of 80% should be covered with indents to ensure high quality spring is produced. Usually, 90 – 100% is the range manufacturers try to aim for. The benefits of shot peening are listed as:[iii]

- The spring surface is harder (stronger)
- The original wire surface is smoother with the original wire drawing marks being largely obliterated.
- There will be a residual compressive stress on the inside surface of the compression spring where the applied stress in service will be a maximum.
-

As it can be seen, in terms of the helical coil compression spring, all the above 3 reasons are absolutely necessary to ensure that we manufacture a safe to use spring for the suspension of the Toyota RAV4 where this spring is being used. These considerably improve the fatigue performance of the spring.

Another quirk added to the spring when manufacturing it is presetting, which is supposed to increase the spring's tensile strength by 75% of the material being used. This is done by making the spring slightly long that its required free length and then compressed to solid. This allows the spring to return to original length when it solidifies. If a stress of more than maximum stress is applied, the spring would permanently deform and would not come back to the original desired length.[iv] The upmost reason for presetting is the increases the load carrying capacity of the spring in fatigue loading. However, this percentage increase does depend on the material and the amount of strain beyond the yield stress.[v]

Toyota RAV4 is the car we are analysing and it operates in numerous regions of the world. In some places in Asian and African subcontinents, the temperatures could reach to as high as 53°C. This is the time especially when people prefer to use their cars to commute. In such harsh condition with high temperatures, there are obvious effects that may or may not be noticeable on the helical coil compression spring that we are analysing. We are aware that as temperature increases, a metallic material becomes more ductile, that is the yield and the tensile strength decreases. Due to this, we would expect the spring constant of the spring to become smaller and, therefore, we should observe a greater displacement from the mean also. We have to take all these factors into consideration when carrying out a fatigue analysis.

Stress endurance limit (S_e') is defined as "the maximum completely reversing cyclic stress that a material can withstand for indefinite (or infinite) number of stress reversals."[vi] This is the maximum alternating strength at which the mechanical component has an infinite life. However, the actual endurance stress is experimentally found to be slightly less than this actual value found. This is why S_e' is multiplied by a factor to adjust to this factor (S_e). Because of this, the actual endurance limit is slightly lower than that found through theory. This is a slightly more conservative approach towards the endurance limit.

In these journal papers found (*Effect of surface finish on Fatigue Strength* by Mohamed Bayoumi[vii] and *Effects of microstructure and surface roughness on the fatigue strength of high-strength steels* by Junbio Lai[viii]) , the authors suggest a heavy dependence of endurance limit on surface roughness. They have proposed various models to assign these. If there is a surface roughness, it already creates the surface cracks that lead to fatigue mechanisms and failure can happen easily while testing. Therefore, the smoother the surface, the higher the endurance limit is predicted to be.

$$S_e = C_L C_G C_S C_T C_R S_e' \qquad \text{Equation 5.1-3}$$

Where C_L is the load factor, C_G is the gradient factor, C_S is the surface factor, C_T is the temperature factor and C_R is the reliability factor. All of these factors vary from material to material. In our case, we are going to assume that of these factors are 1 as specified by the project description. Therefore, the endurance limit is:

$$S_e = S_e' \quad \text{Equation 5.1-4}$$

As mentioned earlier, endurance limit changes as we consider different number of cycles. When we consider 1000 cycle, the endurance limit of the spring will be much higher because in comparison to a million cycles, the spring can withstand less cycles far more easily. As we will be considering different number of cycles, we have to develop a notation for the various cycles. These are listed in the table

Table 5.1-3: Cycle notation

Category	Cycle count
L3	1,000
L4	10,000
L5	100,000
L6 – infinite life	1,000,000

Because of these different cycles, the endurance limit is different for various cycles and they are listed in the table 5.1-4 below. These have been determined through various tedious experiments.

Table 5.1-4: Endurance limit stress for various cycles

	Parameter	Value	Metric Units	Value	English Units
L3s	$S_{3us} = 0.90\ S_{us}$	1152	MPa	167.08	ksi
L4s	$S_{4us} = 0.70\ S_{us}$	896	MPa	129.95	ksi
L5s	$S_{5us} = 0.50\ S_{us}$	640	MPa	92.82	ksi
L6s	$S_{6us} = 0.36\ S_{us}$	464	MPa	67.31	ksi

Using the above parameters, we can finally begin with fatigue analysis by using the Goodman Fatigue Model specified in the next section.

Section 5.2: Modified Goodman Fatigue Model

Goodman fatigue model is superimposed on top of the max-min and alt-mean graphs that we plotted in the previous chapter and allows us to observe the safety factors and find the life expectancy of the spring. However, we are going to use the Modified Goodman Fatigue Model (MGFM). There is a key difference between the modified and normal model and this is dependent on the last part of the previous section. In the MGFM, we consider the endurance limit of the spring for various cycles more realistically/conservatively. This is done by adding various constants, which we observed for our case are all 1, therefore, our modified and normal Goodman models are exactly the same. We are making this assumption because it is easier to carry out our analysis like that.

A MGFM for L3 Case in English units has been plotted below to show exactly how the MGFM figure looks like.

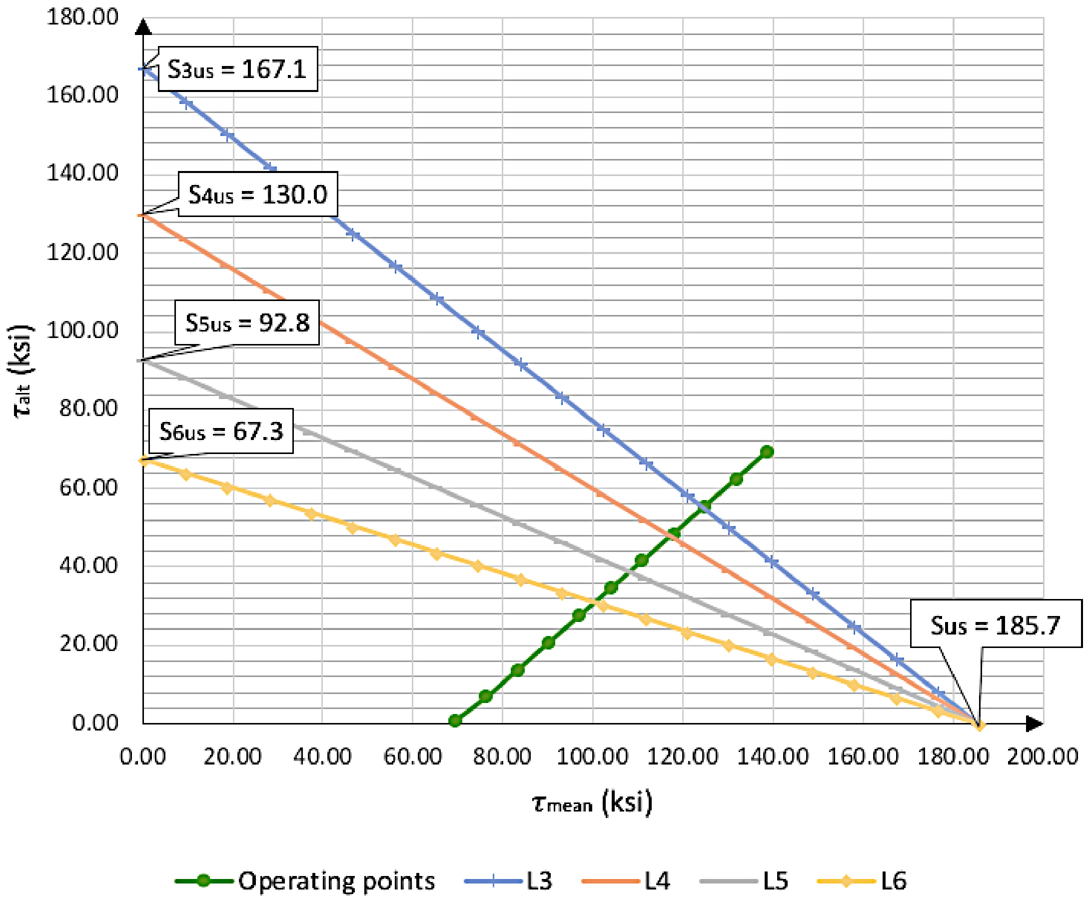

Figure 5.2-1: MGFM plot in ksi for alt-mean fatigue plot

The domain of this graph is from 0 to S_{us} identified earlier and the range is from S_{Nus} to 0. The formula to plot the individual fatigue model is:

$$\tau_{alt-N} = -\frac{S_{Nus}}{S_{us}}\tau_{mean} + S_{Nus} \qquad \text{Equation 5.2-1}$$

In the above equation 5.2-1, N is the fatigue cycle we are interested in finding. This equation works for both the metric and English units. To provide an example using this equation, we have provided the equation for L6 as

$$\tau_{alt-6} = -\frac{S_{6us}}{Sus}\tau_{mean} + S_{6us} \qquad \text{Equation 5.2-2}$$

Similar formulae can be written for L3, L4 and L5 and plotted. In the above figure 5.2-1, these have been plotted. We will save this process for chapter 6 where we do the same in metric units. To explain this graph a little more, we start with the domain of 0 and we increase until we reach the ultimate stress. In the y direction, we start at the endurance stress and slowly decrease to 0. This is because at a mean stress of 0, we can be at the endurance limit, but as we increase the mean stress, we expect the alternate stress to decrease as it becomes harder for the spring to vary as much as it could before.

An equation similar to equation 5.2-1 can be derived also, but before that, let us observe what the actual plot looks like for a max-min graph.

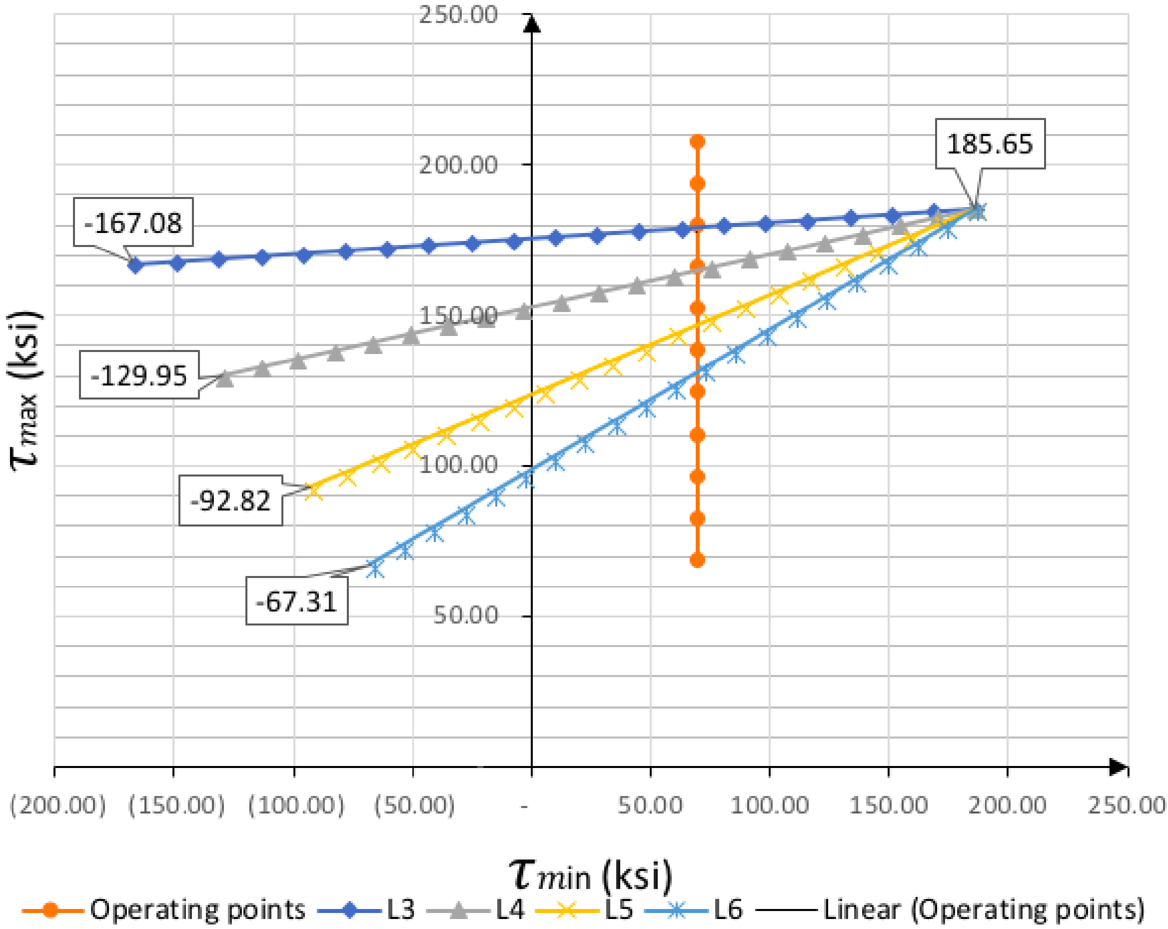

Figure 5.2-2: MGFM in ksi for max-min fatigue plot

Similar to equation 5.2-1, we can derive an equation for max-min fatigue plots also as given in equation 5.2-3.

$$\tau_{max} = S_{Nus} + \frac{S_{us} - S_{Nus}}{S_{us} + S_{Nus}}(S_{Nus} + \tau_{min}) \quad \text{Equation 5.2-3}$$

The above equation has been derived from careful derivation of proportionality. In this figure 5.2-2, since we are aware of the starting and the ending point of the curve, the above equation 5.2-3 is easily to calculate with the use of these two points and by learning the y-axis intercept. The y intercept is found by finding the proportion of indefinite points that lie on the negative x region and the other points that lie on the positive x region. Using this proportionality, the y intercept can be calculated, generally noted by b in linear regression analysis. A sample equation of infinite life has been written below for reference and a better understanding.

$$\tau_{max} = 67.31 + \frac{185.65 - 67.31}{185.65 + 67.31}(67.31 + \tau_{min}) \quad \text{Equation 5.2-4}$$

To explain the figure 5.2-2 a little more, since we are interested in the minimum stress, our endurance stress oscillates from negative to positive every time the spring oscillates. This is why the S_{Nus} starts from negative value and then increases all the way until it reaches a maximum of ultimate tensile shear stress on the right side of the graph. On the range of the graph, it starts at a maximum value of endurance limit as that is range it oscillates at. Following that it increases until it reaches the maximum of ultimate tensile stress. This is what the Modified Goodman Fatigue Model looks like and their associated equations have been listed.

All the parameters needed to plot a MGFM are listed below.

Table 5.2-1: Parameters needed to plot a MGFM

	Parameter	Value	Metric Units	Value	English Units
(S_u)	Ultimate Tensile Stress	1600	MPa	232.06	ksi
(S_{us})	Ultimate Shear Stress	1280	MPa	185.65	ksi
L3s	$S_{3us} = 0.90\ S_{us}$	1152	MPa	167.08	ksi
L4s	$S_{4us} = 0.70\ S_{us}$	896	MPa	129.95	ksi
L5s	$S_{5us} = 0.50\ S_{us}$	640	MPa	92.82	ksi
L6s	$S_{6us} = 0.36\ S_{us}$	464	MPa	67.31	ksi

Using the above parameters as well as the stress operating points from the previous chapter can be compiled to plot a MGFM.

Section 5.3: Fatigue Safety Factor

The fatigue safety factors have been calculated in chapter 3 also. Using a similar approach, we can calculate the safety factor for each of the operating points also. Since both the fatigue plots are trying to achieve the same thing, the number of points with safety factor over 1 are exactly the same but the actual value of the safety factor changes since both the graphs are plotted on different scales.

In order to calculate the safety factor, we have to find the value of the Goodman plot at the x-axis value of the operating point and then divide it by the y value of the fatigue point that we have. A sample calculation for L6 operating point 10 has been carried out in the max-min graph. For operating point 10, the minimum stress is 69.33 ksi and maximum stress is 207.98 ksi. When we use the equation 5.2-3 to find the maximum stress on the Goodman plot at 69.33 ksi, we get 131.23. To find the safety factor, we do the following:

$$\text{SF} = \frac{131.23}{207.98} = 0.63 \qquad \text{Equation 5.3-1}$$

Therefore, the safety factor is 0.63. Since the safety factor is less than 1, we expect the spring to not be able to sustain the load for 10^6 cycles at 3000 lbf. We can do a similar calculation for the alt-mean plot also. To use the same plot point L6 operating point 10, the alternate stress is 138.65 ksi and mean stress is 69.33 ksi. Using the mean stress, we can find the value of the Goodman plot using the equation 5.2-1 to be 17.04. The calculation of the safety factor is given below.

$$\text{SF} = \frac{17.04}{69.33} = 0.25 \qquad \text{Equation 5.3-2}$$

When the results of equation 5.3-1 and 2 are compared, we see that we get a different value for the safety factor. This happens since we are using different scales to calculate each of the following. The only thing we will find in common between the plots is that the number of factors less than 1 will be same for both. This is because they both based on a similar scale.

Therefore, we have seen that in order to calculate the safety factors, we don't really need any new parameters. They already exist, we just have to calculate the different fatigue values for the x-axis value of different life cycles that we look at. These life cycles are listed in the table 5.3-1 below.

Table 5.3-1: Parameters for Safety Factor calculations

Category	Cycle count
L3	1,000
L4	10,000
L5	100,000
L6 – infinite life	1,000,000

We have to calculate the different safety factors for the above mentioned cycles for every operating point. This is a very tedious process and in total 40 safety factors points will be plotted. Out of those, we have to identify which are the factors above the safety factor of 1 and make sure that our helical coil compression spring remains in that particular range of the loading. If it goes above that, the spring is likely to experience failure and the car needs to be changed.

From the previous chapter 4, we are also aware of how long the car needs to run before we reach every number of cycles and using that we can calculate the average life expectancy of the spring and how often it needs to be changed. All of these will be determined in the next chapter 6.

Section 5.4: Table of data

Table 5.4-1: Chapter 5 table of data

Notation	Definition	Metric value	Units	English value	Units
(S_u)	Ultimate Tensile Stress	1600	MPa	232.06	ksi
(S_y)	Tensile Yield Stress	1440	MPa	208.85	ksi
(S_{us})	Ultimate Shear Stress	1280	MPa	185.65	Ksi
(S_{ys})	Shear Yield Stress	831	MPa	120.58	ksi
L4s	$S_{4us} = 0.70\ S_{us}$	896	MPa	129.95	ksi
L5s	$S_{5us} = 0.50\ S_{us}$	640	MPa	92.82	ksi
L6s	$S_{6us} = 0.36\ S_{us}$	464	MPa	67.31	ksi

Table 5.4-2: Chapter 5 table of data life cycles

Category	Cycle count
L3	1,000
L4	10,000
L5	100,000
L6 – infinite life	1,000,000

Section 5.5: References

[i] "Difference Between Yield Strength and Tensile Strength." Pediaa.Com, 14 Oct. 2015, pediaa.com/difference-between-yield-strength-and-tensile-strength/.
[ii] Lehmhus, Dirk. "Why Is Shear Strength Less than Tensile Strength?" Research Gate, www.researchgate.net/post/Why_is_shear_strength_less_than_tensile_strength.
[iii] Hayes, Mark. "Cautionary Tale: Shot Peening Compression Springs." Shot Peening Springs , www.acewirespring.com/manufacturing_industry_shot_peening_compression_springs.html.
[iv] "Operating Stress." Spring Design, Engineering, Manufacturing, www.springhouston.com/products/compression-springs/operating-stress.html.
[v] Atterbury, T. J., and W. B. Diboll. "The Effect of Presetting Helical Compression Springs." Journal of Engineering for Industry, vol. 82, no. 1, 1960, p. 41., doi:10.1115/1.3662990.
[vi] "Design for Cyclic Loading." New Jersey Institute of Technology, web.njit.edu/~sengupta/met%20301/cyclic_loading%20indefinite.pdf.
[vii] Effect of surface finish on Fatigue Strength
[viii] Lai, Junbiao, et al. "Effects of Microstructure and Surface Roughness on the Fatigue Strength of High-Strength Steels." Procedia Structural Integrity, vol. 2, 2016, pp. 1213–1220., doi:10.1016/j.prostr.2016.06.155.

Section 5.6: Level of Effort

I have spent about 15 hours working on this chapter. Thank you.