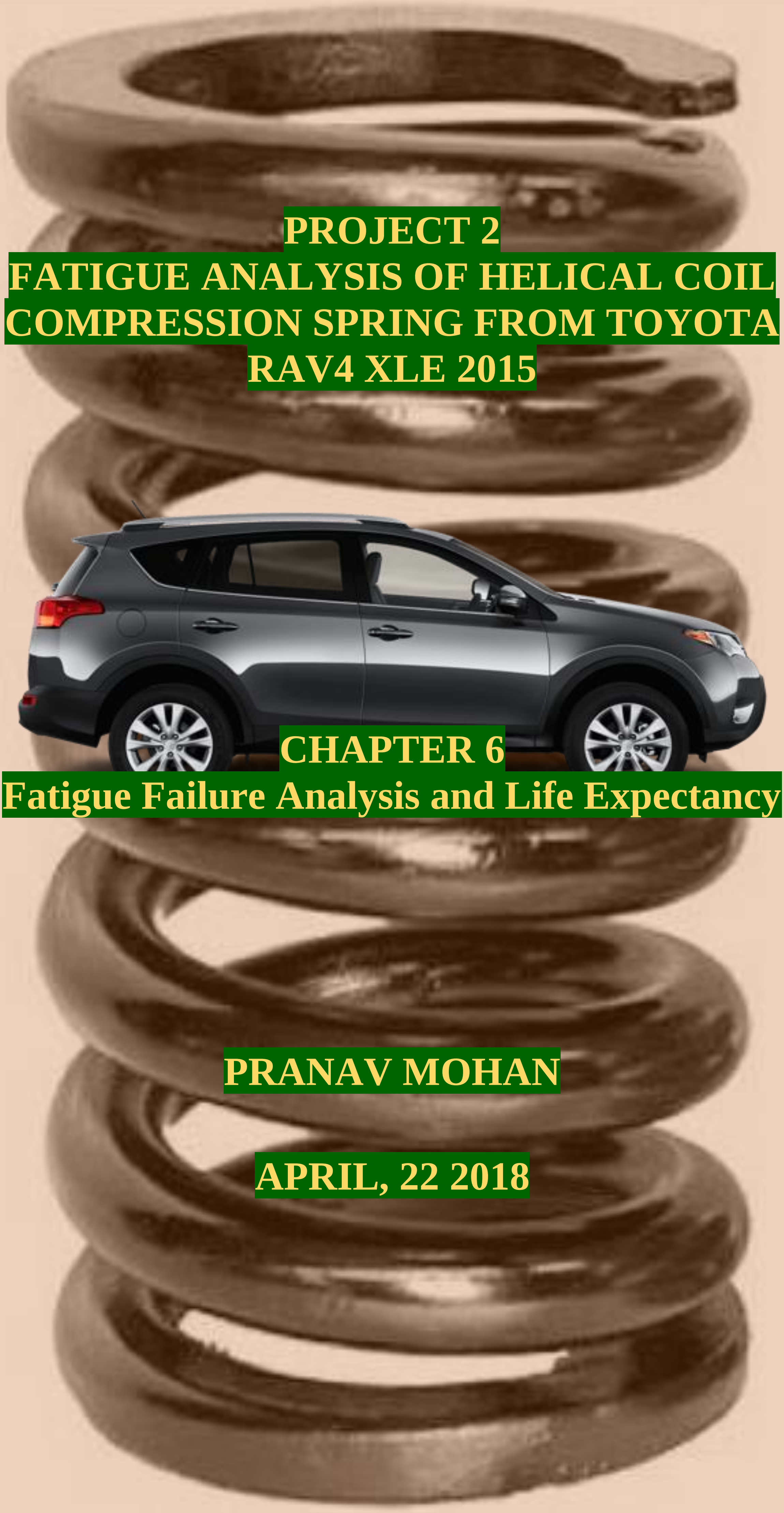

PROJECT 2
FATIGUE ANALYSIS OF HELICAL COIL COMPRESSION SPRING FROM TOYOTA RAV4 XLE 2015

CHAPTER 6
Fatigue Failure Analysis and Life Expectancy

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Table of Content

Chapter 6: Fatigue Failure Analysis and Life Expectancy

All the previous chapters have been set up to come to this chapter. In this chapter, our goal is to create the fatigue plots using the information from chapter 5 and then continue with our fatigue analysis. In doing this, we will find the life expectancy of the spring and calculate the respective safety factor of each of the life cycles that we are interested in.

Section 6.1: Modified Goodman Fatigue Model and Equations

In the chapter 5, we have derived all the equations for the Modified Goodman Fatigue Model (MGFM). These equations, as mentioned earlier, are crucial in creating a fatigue plot and performing a fatigue analysis. This is a good time to define what fatigue means. Fatigue failure is defined as the tendency of a material to fracture by means of progressive brittle cracking under repeated alternating or cyclic stresses of an intensity considerably below the normal strength.[i] As mentioned in chapter 3, we perform cyclical shear stress on the helical coil compression spring and carry out the fatigue analysis for various cycles in terms of L. These have been specified in the table 6.1-1 below.

Table 6.1-1: Cycle notation

Category	Cycle count
L3	1,000
L4	10,000
L5	100,000
L6 – infinite life	1,000,000

In chapter 5, we came up with the equation for all of these life cycles that allows us to plot MGFM on a mean-alt graphs. This equations was:

$$\tau_{alt-N} = -\frac{S_{Nus}}{S_{us}}\tau_{mean} + S_{Nus} \qquad \text{Equation 6.1-1}$$

In this equation S_{Nus} is the shear endurance limit for the associated life cycle that we are working with and S_{us} is the shear stress for the spring that we have specified in chapter 2. Please visit table 5.4-1 and 2.4-1 to be find these specific values. These values have been specified again the table 6.1-2 below.

Table 6.1-2: Endurance limit stress for various cycles

	Parameter	Value	Metric Units	Value	English Units
L3s	$S_{3us} = 0.90\ S_{us}$	1152	MPa	167.08	ksi
L4s	$S_{4us} = 0.70\ S_{us}$	896	MPa	129.95	ksi
L5s	$S_{5us} = 0.50\ S_{us}$	640	MPa	92.82	ksi
L6s	$S_{6us} = 0.36\ S_{us}$	464	MPa	67.31	ksi

For the alt mean graph, we observe that the domain is from 0 to S_{us} and the range is from 0 to S_{Nus}. This equation is generic for all units, as long as the parameters being used are all in the same units.

In chapter 5, we also came with the MGFM plot equation for max-min graph. This equation was:

$$\tau_{max} = S_{Nus} + \frac{S_{us} - S_{Nus}}{S_{us} + S_{Nus}}(S_{Nus} + \tau_{min}) \qquad \text{Equation 6.1-2}$$

Different variables have already been specified in the previous paragraph and tables in the previous chapters. The domain of equation 6.1-2 is from -S_{Nus} to S_{us} and the range is from S_{Nus} to S_{us}. As it can be seen, the range is much smaller in comparison to the domain, and this will be observable in the graph we plot in section 6.4. Once again, this equation generated is applicable to all units.

Section 6.2: Min – Max Fatigue Plots (ksi units and MPa units)

In this section, we can use the equation 6.1-2 generated above to plot the fatigue lines on the operating. The basic plot with only the operating points have been shown in the figure 4.3-4 and 5. Please review that section to understand where those points arise from. Using the MGFM, we can superimpose the fatigue plots on the operating points to observe, which points are under and over the curve. For all the associated τ_{min} values, associated τ_{max} values have been calculated and presented in the table 6.2-1 below.

Table 6.2-1: MGFM fatigue plot data in min-max axis in ksi

N	τ_{min}	τ_{min}	L3 (ksi)		L4 (ksi)		L5 (ksi)		L6 (ksi)	
			τ_{min}	τ_{max}	τ_{min}	τ_{max}	τ_{min}	τ_{max}	τ_{min}	τ_{max}
0	69.3	69.3	-167.1	167.1	-130.0	130.0	-92.8	92.8	-67.3	67.3
1	69.3	83.2	-131.8	168.9	-98.4	135.5	-65.0	102.1	-42.0	79.1
2	69.3	97.1	-96.5	170.8	-66.8	141.1	-37.1	111.4	-16.7	91.0
3	69.3	110.9	-61.3	172.7	-35.3	146.7	-9.3	120.7	8.6	102.8
4	69.3	124.8	-26.0	174.5	-3.7	152.2	18.6	130.0	33.9	114.6
5	69.3	138.7	9.3	176.4	27.9	157.8	46.4	139.2	59.2	126.5
6	69.3	152.5	44.6	178.2	59.4	163.4	74.3	148.5	84.5	138.3
7	69.3	166.4	79.8	180.1	91.0	168.9	102.1	157.8	109.8	150.1
8	69.3	180.2	115.1	181.9	122.5	174.5	130.0	167.1	135.1	162.0
9	69.3	194.1	150.4	183.8	154.1	180.1	157.8	176.4	160.4	173.8
10	69.3	208.0	185.7	185.7	185.7	185.7	185.7	185.7	185.7	185.7

Using these above points, we can plot the fatigue curves on the operating points.

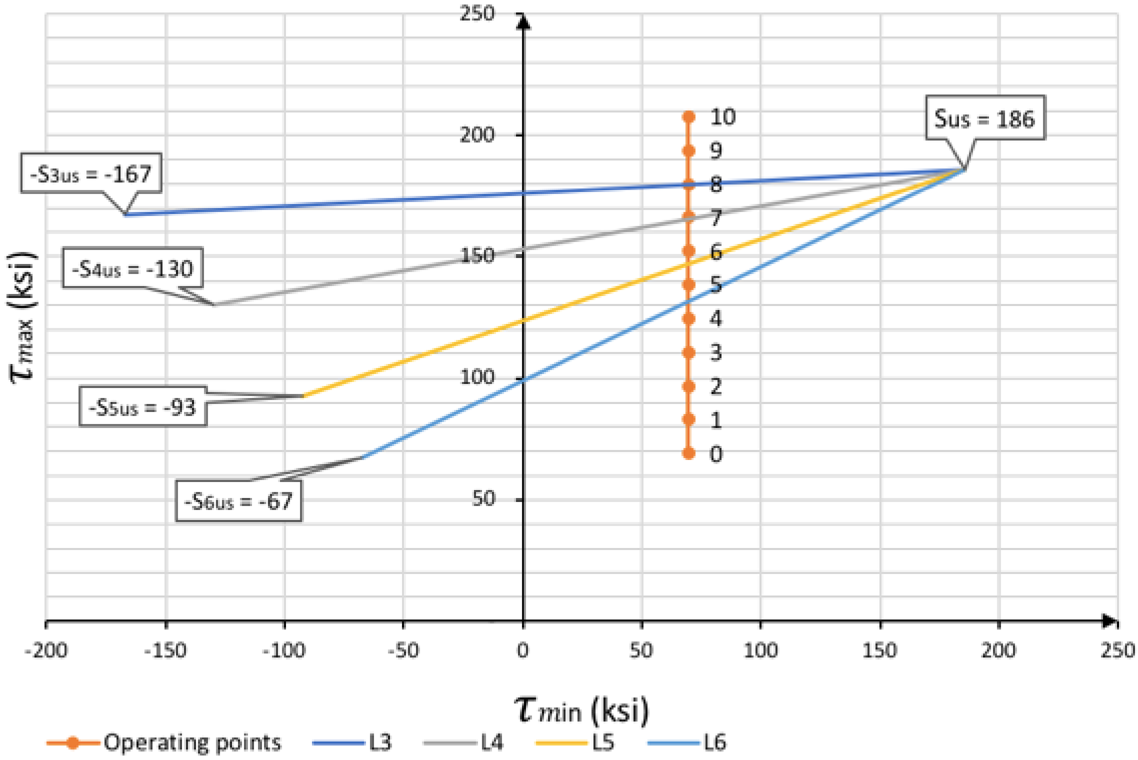

Figure 6.2-1: MGFM plot in ksi

Using this figure, the points under the fatigue curves are considered safe, however, the points that particular are above the curve are with safety factors below 1 and are in danger of failure and we need to careful that the loading on the car is never more than that. We can draw a similar graph for metric units also. The table with the associated points is shown below.

Table 6.2-2: MGFM fatigue plot data in min-max axis in MPa

N	τ_{min}	τ_{min}	L3 (MPa)		L4 (MPa)		L5 (MPa)		L6 (MPa)	
			τ_{min}	τ_{max}	τ_{min}	τ_{max}	τ_{min}	τ_{max}	τ_{min}	τ_{max}
0	478	478	-1152	1152	-896	896	-640	640	-464	464
1	478	574	-909	1165	-678	934	-448	704	-290	546
2	478	669	-666	1178	-461	973	-256	768	-115	627
3	478	765	-422	1190	-243	1011	-64	832	59	709
4	478	860	-179	1203	-26	1050	128	896	234	790
5	478	956	64	1216	192	1088	320	960	408	872
6	478	1,052	307	1229	410	1126	512	1024	582	954
7	478	1,147	550	1242	627	1165	704	1088	757	1035
8	478	1,243	794	1254	845	1203	896	1152	931	1117
9	478	1,338	1037	1267	1062	1242	1088	1216	1106	1198
10	478	1,434	1280	1280	1280	1280	1280	1280	1280	1280

Using the above points, we can further plot them to visualize the fatigue points that we are interested in.

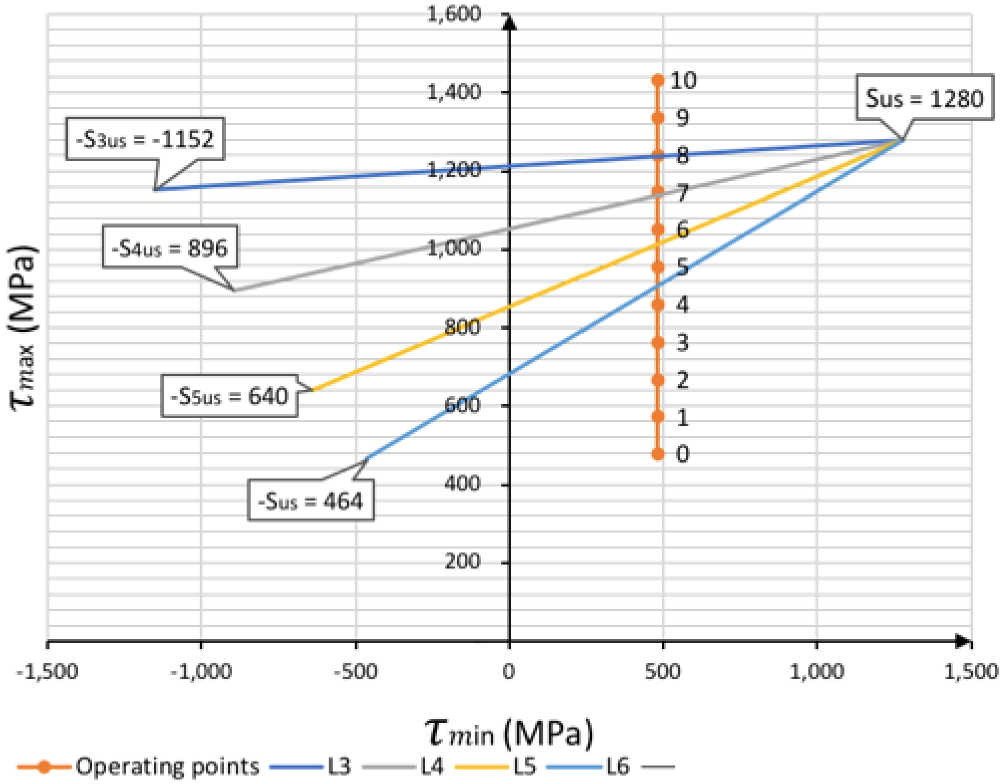

Figure 6.2-2: MGFM plot in max-min axis in Metric units

Once again, we can deduce from the graph the safety factors and which points we should avoid to prevent the spring from failure.

Section 6.3: Mean-Alt Fatigue Plots (ksi units and MPa units)

In this section, we can use the equation 6.1-1 generated in chapter 5 and section 6.1 above we can plot the fatigue lines on the operating. For all the associated τ_{mean} values, associated τ_{alt} values have been calculated and presented in the table 6.2-1 below. In the min-alt graph, all the life cycles have the same domain therefore τ_{mean} has only been shown once in the table 6.3-1 below for English units.

Table 6.3-1: MGFM fatigue plot data in mean-alt axis in ksi

N	τ_{mean} (ksi)	L3	L4	L5	L6
		τ_{alt} (ksi)	τ_{alt} (ksi)	τ_{alt} (ksi)	τ_{alt} (ksi)
0	0.0	167.1	130.0	92.8	67.3
2	18.6	150.4	117.0	83.5	60.6
4	37.1	133.7	104.0	74.3	53.8
6	55.7	117.0	91.0	65.0	47.1
8	74.3	100.2	78.0	55.7	40.4
10	92.8	83.5	65.0	46.4	33.7
12	111.4	66.8	52.0	37.1	26.9
14	130.0	50.1	39.0	27.8	20.2
16	148.5	33.4	26.0	18.6	13.5
18	167.1	16.7	13.0	9.3	6.7
20	185.7	0.0	0.0	0.0	0.0

Using the above points, we can plot the MGFM plot shown in the figure 6.3-1 below.

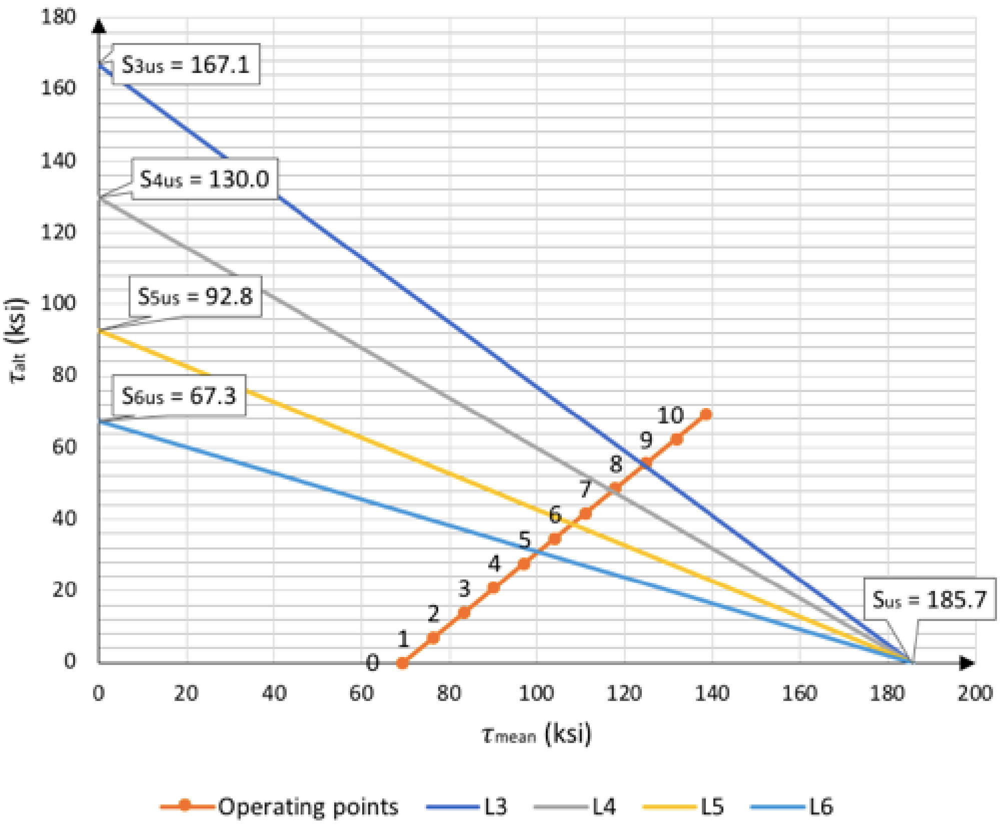

Figure 6.3-1: MGFM plot in alt-mean axis in English units

As mentioned earlier, the points that are under the curve are safe from fatigue, however, the points over the fatigue lines added are in danger of failure at that number of cycles. We observe that for L3, we have three points over the curve, for L4, four points are over the curve, etc. Such analysis has to be carried out for all the lines. We can plot the above graph in the metric units, MPa also. The data to plot the graph is given in the table 6.3-2 below.

Table 6.3-2: MGFM fatigue plot data for mean-alt plot in MPa

N	τ_{mean} (MPa)	L3	L4	L5	L6
		τ_{alt} (MPa)	τ_{alt} (MPa)	τ_{alt} (MPa)	τ_{alt} (MPa)
0	0	1152	896	640	464
2	128	1037	806	576	418
4	256	922	717	512	371
6	384	806	627	448	325
8	512	691	538	384	278
10	640	576	448	320	232
12	768	461	358	256	186
14	896	346	269	192	139
16	1024	230	179	128	93
18	1152	115	90	64	46
20	1280	0	0	0	0

Using the above table 6.3-2, we can plot the MGFM graph in mean-alt axis in metric units.

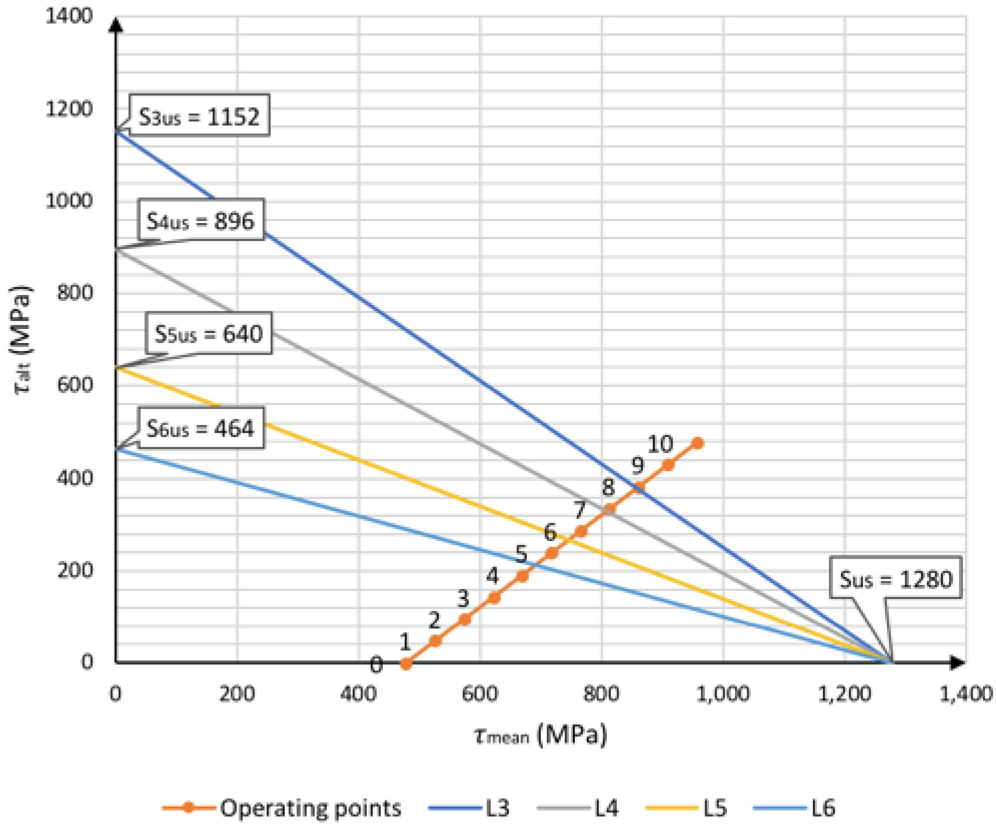

Figure 6.3-2: MGFM plot in mean-alt axis in metric units

Using the above 4 graphs that we have plotted in the previous two sections, we can calculate the life expectancy of the spring with varying loads. This is the motive of the coming section where we find out the appropriate force needed to sustain at the specified life cycle.

Section 6.4: Life Expectancy

Life expectancy of the helical coil compression spring is defined as the average age of the spring where we will expect the spring to face failure (shown in figure 5.1-1) at the top of the graph. The spring goes through numerous cycles and by using the associated fatigue models, we can confirm the life expectancy of the spring. As mentioned in chapter 2 and chapter 4, we are making an assumption that our car runs at 10,000 miles an year and there are about 10 cycles every mile. Using these two statistics about the car, we can come up with the life and loading for each cycle for the spring.

Table 6.4-1: N loading for the points at which the spring will fail

# of cycles	Life cycle	# of years	N value	Max Loading (lbs)
1000	L3	0.01	7	2400
10000	L4	0.1	6	2200
100000	L5	1	5	2000
1000000	L6	10	4	1800

In the above table, we have identified the number of points that are under the curve and based on those, we have calculated the maximum loading possible using the equation 4.1-1, where it can be for the various loading conditions that we had identified. The N value can be identified by looking under the curve of any of the above 4 figures presented in the previous two sections.

We want to quantify the above N value using the safety factors, which are presented in the coming sections in terms of the safety factors.

Section 6.5: Fatigue Safety Factors for L3s Life

The method to find the fatigue safety factors has been discussed in the section 5.3. Please visit to learn how to calculate these. The method has been briefly described here. To find the fatigue safety factor of the min-max graph, first find the intersection of the operating points line with the MGFM line, then subtract the preload torque from the y value. Then divide the quantity found by the y value of the operating point minus the pre-load torque. Similarly, the graph for mean-alt graph, find the intersection of the operating points line and the MGFM plot. Then divide the y value by the y value of the operating point to find the safety factor. The interesting part to know is that **the values of the safety factors are exactly the same in both the plots and the units.** For L3, the fatigue the point of intersection for min-max is (69.33, 179.53 ksi) and for mean-alt plot is (124.43, 55.10 ksi). Using these points, we can find the safety factors for the L3 life cycle.[ii]

Table 6.5-1: Safety Factors for L3

Operating points	L3
1	7.95
2	3.97
3	2.65
4	1.99
5	1.59
6	1.32
7	1.14
8	0.99
9	0.88
10	0.79

For L3 life, we see that 7 points are above the range of 1 and are safe. So this means that for 1000 cycles, a weight of 2400 lbs can be applied on the spring and there is no danger of failure in this time.

Section 6.6: Fatigue Safety Factors for L4s Life

The process to find the safety factors for the L4 life is very similar to that of L3. The only difference is that the point of intersection in both the figures is different. For L4, the fatigue the point of intersection for min-max is (69.33, 165.12 ksi) and for mean-alt plot is (117.23, 47.90 ksi). Using these points, we can find the safety factors for the L4 life cycle.

Table 6.6-1: Safety Factors for L4

Operating points	L4
1	6.91
2	3.45
3	2.30
4	1.73
5	1.38
6	1.15
7	0.99
8	0.86
9	0.77
10	0.69

For L4 life, we see that 6 points are above the range of 1 and are safe. So this means that for 10000 cycles, a weight of 2200 lbs can be applied on the spring and there is no danger of failure in this time.

Section 6.7: Fatigue Safety Factors for L5s Life

Once again the method to find the L5 fatigue safety factors is similar to the methods before. For L5, the fatigue the point of intersection for min-max is (69.33, 146.87 ksi) and for mean-alt plot is (108.10, 38.77 ksi). Using these points, we can find the safety factors for the L5 life cycle.

Table 6.7-1: Safety Factors for L5

Operating points	L5
1	5.59
2	2.80
3	1.86
4	1.40
5	1.12
6	0.93
7	0.80
8	0.70
9	0.62
10	0.56

For L5 life, we see that 5 points are above the range of 1 and are safe. So this means that for 100000 cycles, a weight of 2000 lbs can be applied on the spring and there is no danger of failure in this time.

Section 6.8: Fatigue Safety Factors for L6s Life

L6 is 1,000,000 cycles, which is also known as the infinite cycles. I car covers 1 million cycles in 10 years, according to table 6.4-1. Finally, we can calculate the safety factors of infinite cycles also. For L6, the fatigue the point of intersection for min-max is (69.33, 131.23 ksi) and for mean-alt plot is (100.28, 30.95 ksi). Using these points, we can find the safety factors for the L6 life cycle.

Table 6.8-1: Safety Factors for L6

Operating points	L6
1	4.46
2	2.23
3	1.49
4	1.12
5	0.89
6	0.74
7	0.64
8	0.56
9	0.50
10	0.45

For infinite life, we see that 4 points are above the range of 1 and are safe. So this means that for 1000000 cycles, a weight of 1800 lbs can be applied on the spring and there is no danger of failure in this time. This is much smaller in comparison to the maximum of 3000lbs that was applied.

What are the implications of knowing the safety factor? We have learnt from our analysis the maximum weight allowed if we want our car to survive for the specified life span of a car. If a user of Toyota RAV4 XLE is always going around with weight more than 1800 lbs, which is with the family, then it is suggested that the user must change the helical coil suspension spring within 10 years of purchase of the car. If the user generally drives alone, which is less than 1600lbs, then the user does not need to worry about the spring suspensions.

Section 6.9: Table of data

Table 6.9-1: Cycle notation

Category	Cycle count
L3	1,000
L4	10,000
L5	100,000
L6 – infinite life	1,000,000

Table 6.9-2: Endurance limit stress for various cycles

	Parameter	Value	Metric Units	Value	English Units
L3s	$S_{3us} = 0.90\ S_{us}$	1152	MPa	167.08	ksi
L4s	$S_{4us} = 0.70\ S_{us}$	896	MPa	129.95	ksi
L5s	$S_{5us} = 0.50\ S_{us}$	640	MPa	92.82	ksi
L6s	$S_{6us} = 0.36\ S_{us}$	464	MPa	67.31	ksi

Table 6.9-3: MGFM fatigue plot data in ksi and MPa

N	τ_{min}	τ_{min}	L3 (ksi)		L4 (ksi)		L5 (ksi)		L6 (ksi)	
			τ_{min}	τ_{max}	τ_{min}	τ_{max}	τ_{min}	τ_{max}	τ_{min}	τ_{max}
0	69.3	69.3	-167.1	167.1	-130.0	130.0	-92.8	92.8	-67.3	67.3
1	69.3	83.2	-131.8	168.9	-98.4	135.5	-65.0	102.1	-42.0	79.1
2	69.3	97.1	-96.5	170.8	-66.8	141.1	-37.1	111.4	-16.7	91.0
3	69.3	110.9	-61.3	172.7	-35.3	146.7	-9.3	120.7	8.6	102.8
4	69.3	124.8	-26.0	174.5	-3.7	152.2	18.6	130.0	33.9	114.6
5	69.3	138.7	9.3	176.4	27.9	157.8	46.4	139.2	59.2	126.5
6	69.3	152.5	44.6	178.2	59.4	163.4	74.3	148.5	84.5	138.3
7	69.3	166.4	79.8	180.1	91.0	168.9	102.1	157.8	109.8	150.1
8	69.3	180.2	115.1	181.9	122.5	174.5	130.0	167.1	135.1	162.0
9	69.3	194.1	150.4	183.8	154.1	180.1	157.8	176.4	160.4	173.8
10	69.3	208.0	185.7	185.7	185.7	185.7	185.7	185.7	185.7	185.7
0	478	478	-1152	1152	-896	896	-640	640	-464	464
1	478	574	-909	1165	-678	934	-448	704	-290	546
2	478	669	-666	1178	-461	973	-256	768	-115	627
3	478	765	-422	1190	-243	1011	-64	832	59	709
4	478	860	-179	1203	-26	1050	128	896	234	790
5	478	956	64	1216	192	1088	320	960	408	872
6	478	1,052	307	1229	410	1126	512	1024	582	954
7	478	1,147	550	1242	627	1165	704	1088	757	1035
8	478	1,243	794	1254	845	1203	896	1152	931	1117
9	478	1,338	1037	1267	1062	1242	1088	1216	1106	1198
10	478	1,434	1280	1280	1280	1280	1280	1280	1280	1280

Table 6.9-4: MGFM fatigue plot data in ksi

N	τ_{mean} (ksi)	L3	L4	L5	L6
		τ_{alt} (ksi)	τ_{alt} (ksi)	τ_{alt} (ksi)	τ_{alt} (ksi)
0	0.0	167.1	130.0	92.8	67.3
2	18.6	150.4	117.0	83.5	60.6
4	37.1	133.7	104.0	74.3	53.8
6	55.7	117.0	91.0	65.0	47.1
8	74.3	100.2	78.0	55.7	40.4
10	92.8	83.5	65.0	46.4	33.7
12	111.4	66.8	52.0	37.1	26.9
14	130.0	50.1	39.0	27.8	20.2
16	148.5	33.4	26.0	18.6	13.5
18	167.1	16.7	13.0	9.3	6.7
20	185.7	0.0	0.0	0.0	0.0
0	0	1152	896	640	464
2	128	1037	806	576	418

4	256	922	717	512	371
6	384	806	627	448	325
8	512	691	538	384	278
10	640	576	448	320	232
12	768	461	358	256	186
14	896	346	269	192	139
16	1024	230	179	128	93
18	1152	115	90	64	46
20	1280	0	0	0	0

Table 6.9-5: N loading for the points at which the spring will fail

# of cycles	Life cycle	# of years	N value	Max Loading (lbs)
1000	L3	0.01	7	2400
10000	L4	0.1	6	2200
100000	L5	1	5	2000
1000000	L6	10	4	1800

Table 6.9-6: Safety Factors

Operating points	L3	L4	L5
1	7.95	6.91	5.59
2	3.97	3.45	2.80
3	2.65	2.30	1.86
4	1.99	1.73	1.40
5	1.59	1.38	1.12
6	1.32	1.15	0.93
7	1.14	0.99	0.80
8	0.99	0.86	0.70
9	0.88	0.77	0.62
10	0.79	0.69	0.56

Section 6.10: References

[i] LLC. "Fatigue - Strength (Mechanics) of Materials." Engineers Edge, www.engineersedge.com/material_science/fatigue_failure.html
[ii] Lecture Notes – AME 3353 – 04-22-2018

Section 6.11: Level of Effort

I spent about 10 hours working on this report. Thank you