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Chapter 3
Pre-Load Conditions on Mechanical Component with FBDs, Statics and Stress Analysis

Project 1
Fatigue Analysis of Wheel Lug Stud/Wheel lug bolt

Vehicle:
Toyota RAV4 XLE 2015

Wheel Lug Stud

AME 3353
Design of Mechanical Components
Professor Harold L. Stalford

3.1 Pre-Load Torque Conditions

The mechanical component being analysed is a lug stud from the front right wheel of Toyota RAV4 XLE 2015. This is a very powerful SUV and therefore, requires very secure torque and clamping force on these studs. For this reason, we are going to analyse the pre-load torque on a lug stud and the lug bolt so that we can learn more about the RAV4 and its performance. Below is a picture of a lug stud (figure 3.1-1).

Figure 3.1-1: Lug stud of Toyota RAV4 XLE 2015

To make the car move, some initial torque needs to be applied, which is called pre-load torque. This is why we cannot push a car to move. This torque can be measured using numerous analogue and digital devices. This information has been found online for RAV4 XLE to be 80 fit-lbs[1] or 108.46Nm. This conversion is given in equation 3.1-1.

$$80 * ft * lbs * \frac{1m}{3.28084ft} * \frac{1N}{.22481lbs} = 108.46Nm \qquad (3.1\text{-}1)$$

This means that the engine of the car needs to deliver at least 80ft-lbs or 108.46Nm to the wheels of the car in total to make it move initially.

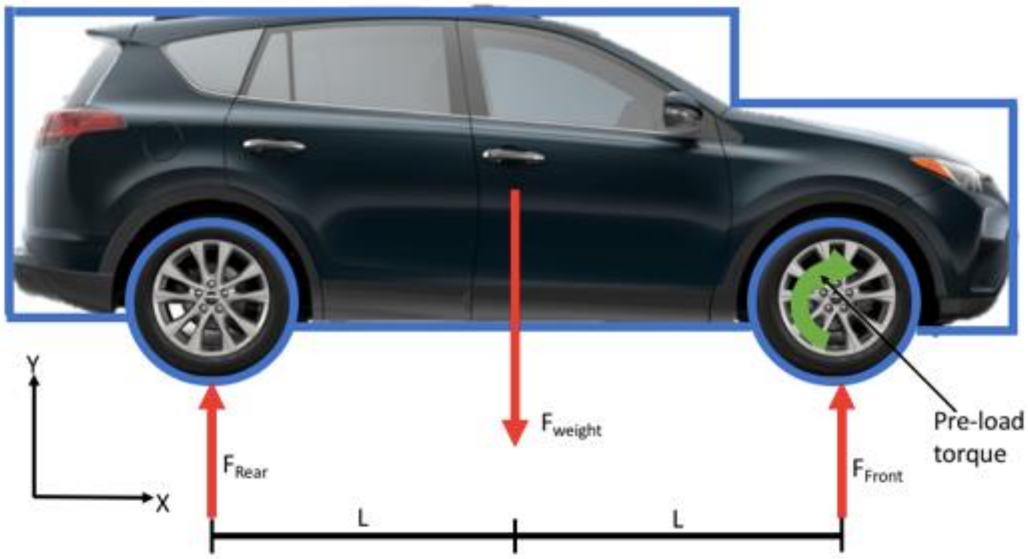

Figure 3.1-2: Free-body diagram of RAV4 with reactions and pre-load Torque aplied

We would like to find the reaction forces (F_{Normal}) for which we need to perform a free body diagram analysis. In this document, all the forces are colored as red and the moments and torques are colored as green. The figure 3.1-2 above shows the free body diagram on RAV4 for clear demonstration.

$$\sum M = 0 \qquad (3.1\text{-}2)$$

We need to find the summation of moments to find the reaction forces. It is to be noticed that there are 2 F_{Front} and 2 F_{Rear} forces for the four wheels in the car.

$$-T_{pre-load} + F_{weight}L - 2F_{Rear} * 2L \quad (3.1\text{-}3)$$

Plugging in the values into the equation 3.1-3. F_{weight} can be calculated as $g * W_{full}$, which is the vehicle full capacity mass times the gravitational acceleration to give the weight of the car.

$$-108.46Nm + 1982kg * 9.81ms^{-2} * 1.461m - 2 * F_{Rear} * 2 * 1.461m = 0 \quad (3.1\text{-}4)$$

The equation 3.1-4 can be solved for F_{Rear} = 4842N. Similarly, F_{Front} can also be found by summing the forces in the y direction (equation 3.1-5) and solving for it (3.1-6).

$$\sum F_y = 0 \quad (3.1\text{-}5)$$

$$-F_{weight} + 2 * F_{Front} + 2 * F_{Rear} = 0 \quad (3.1\text{-}6)$$

$$-1982kg * 9.81ms^{-2} + 2 * F_{Front} + 2 * 4842N = 0 \quad (3.1\text{-}7)$$

Therefore, F_{Front} = 4879N after solving 3.1-7. Furthermore, we need to create a free body diagram (figure 3.1-3) of the lug stud and the log nut to analyse the preload conditions on these mechanical components.

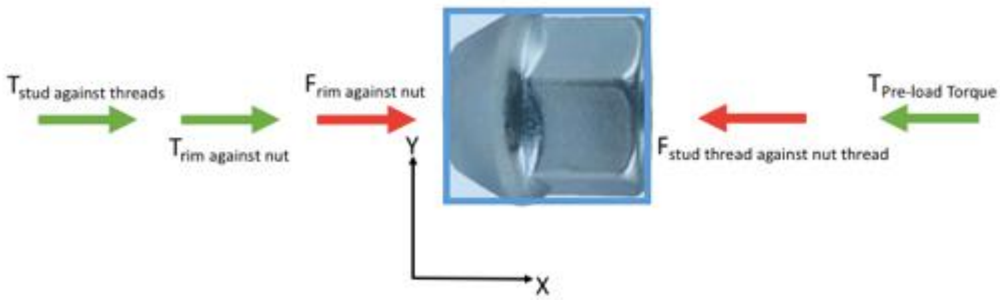

Figure 3.1-3: Free Body diagram of a lug nut

Various forces act on the lug nut. These have been described below:

1. $F_{rim\ against\ nut}$:When the lug nut is resting against the rim, the rim exerts a force on the nut.
2. $F_{stud\ thread\ against\ nut\ thread}$: When the nut is screwed into the stud, the threads of each component exert a force on each other.

Various torques act on the lug nut. These have been described below:

1. $T_{stud\ against\ threads}$: When the nut is screwed into the stud, a torque acts on the nut to resist this motion
2. $T_{rim\ against\ nut}$: When the nut is screwed into the stud, it rests on the rim a torque acts on the nut.
3. $T_{Pre-load\ Torque}$: Pre-load Torque

Similarly, a free body diagram can be drawn for a lug stud (figure 3.1-4).

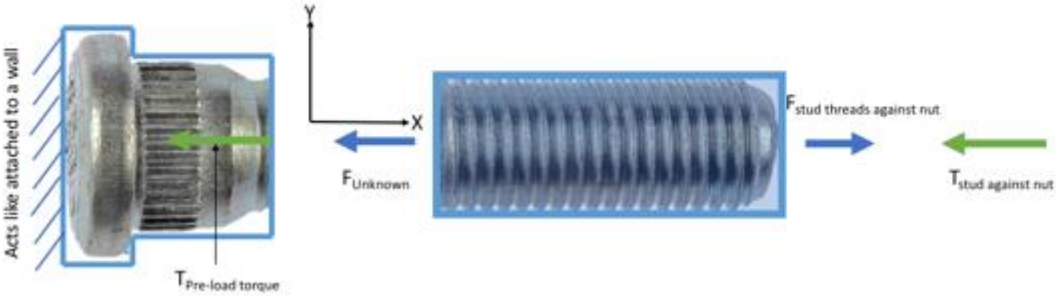

Figure 1-4: Free body diagram of a lug stud

Two forces act on the lug stud. They are described below:

1. $F_{stud\ threads\ against\ nut}$: When the nut is screwed into the stud, the threads exert a force on the threads of the stud
2. $F_{unknown}$: This is force from the threads to the shoulder/collar of the stud.

Similarly, two torques act on the lug stud. They are described below:

1. $T_{stud\ against\ nut}$: When the nut is screwed into the stud, a torque acts on the stud to resist this motion
2. $T_{Pre-load\ torque}$: This is the pre-load torque.

The table below provides all the preload quantities with their units.

Table 3.1-1: Table containing all the pre-load quantities

Parameter	**Description**	**Value in English Units**	**Value in Metric units**
$\boldsymbol{T_{Pre-load}}$	Pre-load Torque	80 ft lbs	108.46 Nm
F_{Rear}	Reaction force at the rear tyres	1088 lbs	4842 N
F_{Front}	Reaction force at the front tyres	1097 lbs	4879 N
$F_{stud\ thread\ against\ nut\ hreads}$	Reaction force of the stud thread the nut threads	TBD Lbs	TBD N
$F_{rim\ against\ nut}$	Reaction force of the rim against the nut	TBD Lbs	TBD N
$T_{stud\ against\ threads}$	Torque of the stud against threads	TBD Ft-lbs	TBD Nm
$\boldsymbol{T_{rim\ against\ nut}}$	**Torque of the rim against the nut**	TBD Ft-lbs	TBD Nm
$\boldsymbol{F_{unknown}}$	**Reaction force of the body to the shoulder of the lug stud**	TBD Lbs	TBD N
$\boldsymbol{F_{stud\ threads\ against\ nut}}$	**Reaction force of the stud against the nut**	TBD Lbs	TBD N
$\boldsymbol{T_{stud\ against\ nut}}$	**Torque of the stud against the nut**	TBD Ft-lbs	TBD Nm

3.2 Pre-Tensile Load

The pre-loaded torque is found to be 80 ft lbs and this is the torque we will place on the stud. In this section, we are going to determine the clamping force of the rim on hub due to total pre-tensile load on lug stud. For this, we will need moment and force diagrams on the free body diagrams.

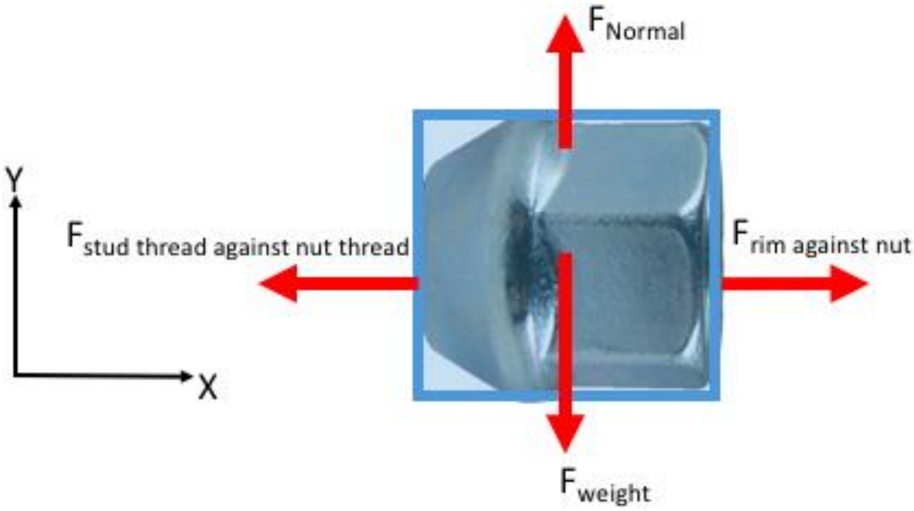

Figure 3.2-1: Forces on a lug nut before loading

Using the figure 3.2-1 above, we can write the summation of forces in the x (3.2-4 – 6) and y (3.2-1 – 3) direction.

$$\sum F_y = 0 \tag{3.2-1}$$

$$F_{normal} - F_{weight} = 0 \tag{3.2-2}$$

$$F_{normal} = F_{weight} \tag{3.2-3}$$

$$\sum F_x = 0 \tag{3.2-4}$$

$$-F_{stud\ thread\ against\ nut\ thread} + F_{rim\ against\ nut} = 0 \tag{3.2-5}$$

$$F_{stud\ thread\ against\ nut\ thread} = F_{rim\ against\ nut} \tag{3.2-6}$$

Similarly, a free body diagram for the moment can also be created (figure 3.2-2).

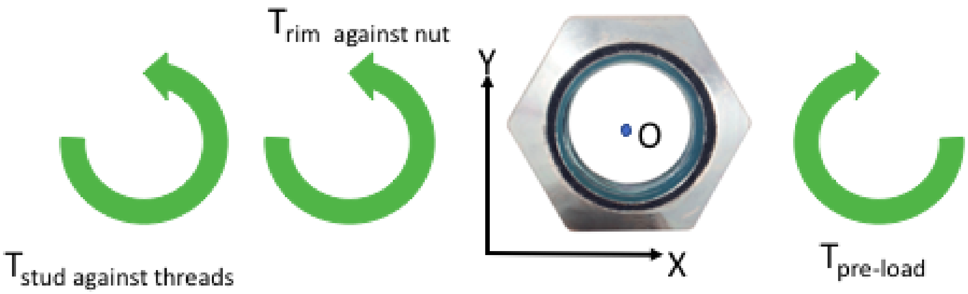

Figure 3.2-2: Moments on a lug nut before loading

Equation of moments (3.2-7 – 8) can be written down for this scenario of this lug nut.

$$\sum M_o = 0 \tag{3.2-7}$$

$$T_{rim\ against\ nut} + T_{stud\ against\ threads} - T_{pre-loaded} = 0 \tag{3.2-8}$$

This lug nut goes into a lug stud and we need to analyse the lug stud also to find the clamping force. Therefore, a free body diagram of the lug stud for all the forces is shown below (figure 3.2-3).

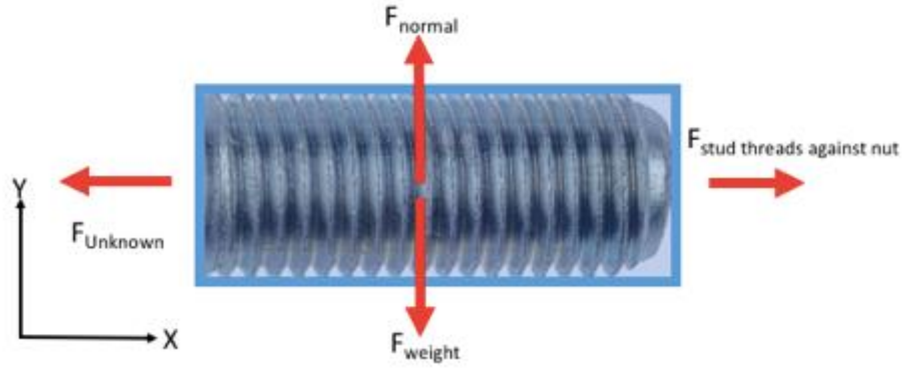

Figure 3.2-3: Forces on a lug stud before loading

The forces can be balanced in the x (3.2-12 – 14) and the y (3.2-9 – 11) direction.

$$\sum F_y = 0 \tag{3.2-9}$$

$$F_{normal} - F_{weight} = 0 \tag{3.2-10}$$

$$F_{normal} = F_{weight} \tag{3.2-11}$$

$$\sum F_x = 0 \tag{3.2-12}$$

$$-F_{stud\ thread\ against\ nut} + F_{unknown} = 0 \tag{3.2-13}$$

$$F_{unknown} = F_{rim\ against\ nut} \tag{3.2-14}$$

This Unknown force can also be represented as the tension force and will be calculated in the later section. Before that, we need to provide the moment diagram (figure 3.2-4).

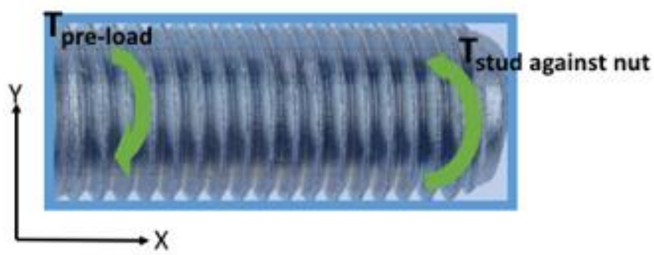

Figure 3.2-4: Moments on a lug stud before loading

Finally, moment equations can also be listed just like before (equation 3.2-15 – 17).

$$\sum M_o = 0 \tag{3.2-15}$$

$$-T_{stud\ against\ nut} + T_{pre-loaded} = 0 \tag{3.2-16}$$

$$T_{stud\ against\ nut} = T_{pre-loaded} \tag{3.2-17}$$

The pre-load torque can be calculated by the following equation 3.2-18. However, we already know the pre-load torque from the research conducted. We need to find out the clamping force represented represented by W.

$$T = \frac{Wd_m}{2}\frac{f\pi d_m + Lcos\alpha_n}{\pi d_m cos\alpha_n - fL} + \frac{Wf_c d_c}{2} \quad (3.2\text{-}18)$$

$$T = \frac{Wd_m}{2}\frac{f\pi d_m + Lcos\alpha_n}{\pi d_m cos\alpha_n - fL} + \frac{Wf_c d_c}{2} * \frac{d_m}{d_m} \quad (3.2\text{-}19)$$

$$T = \frac{Wd_m}{2}\left[\frac{f\pi d_m + Lcos\alpha_n}{\pi d_m cos\alpha_n - fL} + \frac{f_c d_c}{d_m}\right] \quad (3.2\text{-}20)$$

Therefore, we can rearrange the equation 3.2-20 to make clamping force (W) the subject.

$$W = \frac{2T}{d_m\left[\frac{f\pi d_m + Lcos\alpha_n}{\pi d_m cos\alpha_n - fL} + \frac{f_c d_c}{d_m}\right]} \quad (3.2\text{-}21)$$

There are a lot of parameters in the equation 3.2-21 above. Therefore, we will present it in a table to organize it.

Table 3.2-1: Parameters for calculating the clamping force

Parameter	**Description**	**Value in English Units**	**Value in Metric units**
T	Pre load torque	80 ft lbs	108.46 Nm
α_n	Angle	30°	30°
d_m	Mean diameter of thread contact	0.472 in	0.012 m
f	Friction	0.15	0.15
L	Length of the stud	0.059 in	0.015 m
f_c	Collar Friction	0.15	0.15
d_c	Collar diameter	.906 in	0.023 m

Using these values, we calculated the clamping force (W) to be 43,065N or 9681 lbs.

We can also approximate the clamping for using the following equation 3.2-22.

$$W = \frac{T}{0.2 * d} \quad (3.2\text{-}22)$$

The value found using this equation is 45192N or 10160 lbs. Since this is an approximate value, we do not get an exact value but we get very close the real value from equation 3.2-21. Therefore, for Toyota RAV4 XLE 2015 lug studs, this formula does not seem to apply.

This clamping force does not create a shear force because the clamping force acts normal to the lug stud and the wheel. This is shown in figure 3.2-5 below.

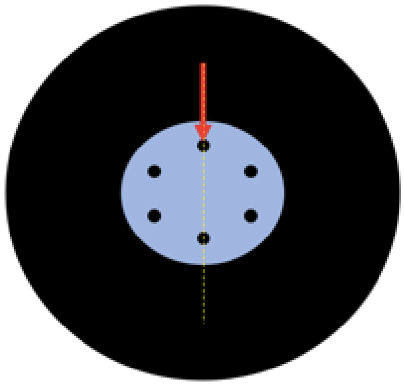

Figure 3.2-5: Clamping force on the lug stud

There is no moment arm, therefore no shear force is created. This is why, the lug studs can survive without fatigue for a very long time. Vehicle breaking and acceleration do not affect the lug stud or the lug nut because a shear force is not created.

3.3 Stress Analysis and Static Safety Factor

Now that we have a value for clamping force, we can calculate the different stresses associated with the lug stud and find the minimum normal, maximum normal and maximum shear stresses within lug/stud. We can calculate the normal axial stress using the following equation 3.3-1.

$$\sigma = \frac{W}{A_m} \tag{3.2-21}$$

The table of parameters for calculating the axial stress can be found below.

Table 3.3-1: Parameters for calculating normal stress

Parameter	Description	Value in English Units	Value in Metric units
W	Clamping force or pre-tensile axial force	4510 lbs	20065N
A_m	Cross-sectional area of the lug stud	0.175in^2	0.000113m^2

Therefore, the normal stress is found to be 177.57 MPa or 25.75 ksi. Now we can calculate the maximum and minimum normal and maximum shear stress using the Mohr's circle. One stress element from the lug stud is represented below in figure 3.3-1.
The value of shear stress can be found from the equation below:

$$\tau = \frac{Tc}{\pi r^4} \tag{3.2-21}$$

Table 3.3-2: Parameters to calculate the shear stress

Parameter	Description	Value in English Units	Value in Metric units
T	Pre load torque	80 ft -lbs	108.46
c,r	Radius of the lug stud	.236 in	.06m

Using the above parameters, shear stress is found to be 0.502 MPa. Since the shear stress is very small in comparison to the normal stress. The figure below shows the stress element for before loading.

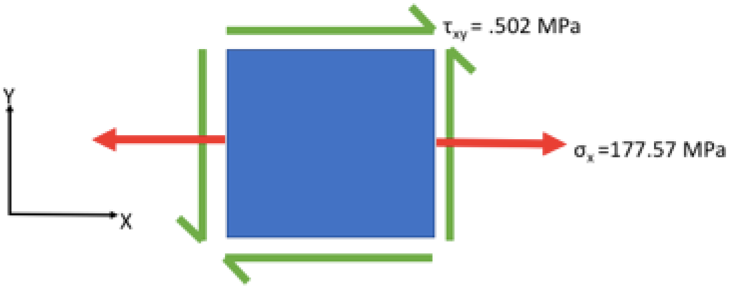

Figure 3.3-1: Stress element before loading

Using these stresses, a Mohr's circle diagram has been created in the following figure 3.3-2.

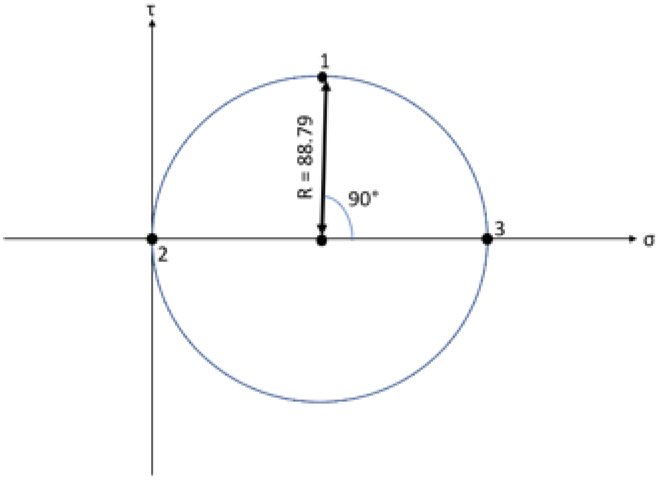

Figure 3.3-2: Mohr's Circle for lug stud before loading

The radius of the circle is 88.79, therefore using this, we can calculate the maximum and minimum normal stress and maximum shear stress. These values are listed in the table below:

Table 3.3-1: Parameters for Mohr's Circle

Number	Parameter	Description	Value in English Units	Value in Metric units
3	σ_{max}	Maximum Normal stress	12.88 ksi	88.79 MPa
2	σ_{min}	Minimum Normal stress	0	0
1	σ_{max}	Maximum Normal stress	12.88 ksi	88.79 MPa

The stress element with the above values is shown below in figure 3.3-3.

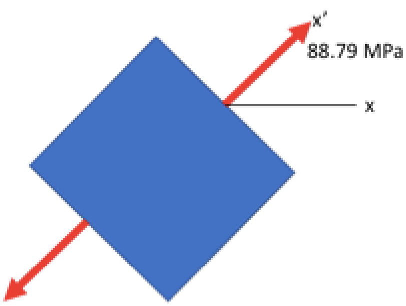

Figure 3.3-3: Stress element after orientation

Because the shear stress is so small, it doesn't have a very big effect.

"During initial use, the screw or bolt usually "unwinds" very slightly, relieving most or all of the torsion" from page 434 in mentions that the small shear stresses are relieved after the car starts moving and therefore, shear stresses are not required in fatigue analysis[2].

After the car starts moving, the stress element looks like this figure 3.3-4.

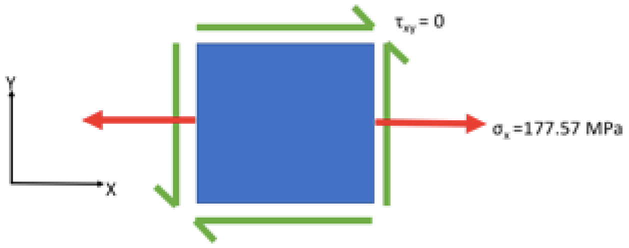

Figure 3.3-4: Stress element after pre-loading

The ultimate yield stress of the lug stud is 940 MPa and what we have found is only 177.57 MPa. Therefore, the safety factor can be calculated to 5.23, which is really safe and is realistic.

3.4 Table of Data and Results

Table 3.4-1: All the parameters from this chapter

Parameter	Description	Value in English Units	Value in Metric units
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A_m	Cross-sectional area of the lug stud	$0.175in^2$	$0.000113m^2$
T	Pre load torque	80 ft -lbs	108.46
c,r	Radius of the lug stud	.236 in	.06m

σ_{max}	Maximum Normal stress	12.88 ksi	88.79 MPa
σ_{min}	Minimum Normal stress	0	0
σ_{max}	Maximum Normal stress	12.88 ksi	88.79 MPa

3.5 References

[1] *Discounttiredirect.com*, www.discounttiredirect.com/learn/wheel-torque.

[2] Robert C. Juvinall (Author) › Visit Amazon's Robert C. Juvinall Page Find all the books. "Fundamentals of Machine Component Design 5th Edition." *Fundamentals of Machine Component Design: Robert C. Juvinall, Kurt M. Marshek: 9781118012895: Amazon.com: Books*, www.amazon.com/Fundamentals-Machine-Component-Design-Juvinall/dp/1118012895.

3.6 Level of Effort

Doing this project almost took my 25 hours to finish. I really enjoyed it but it was really tedious.