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Chapter 4
Pre-Load Conditions on Mechanical Component with FBDs, Statics and Stress Analysis

Project 1
Fatigue Analysis of Wheel Lug Stud/Wheel lug bolt

Vehicle:
Toyota RAV4 XLE 2015

Wheel Lug Stud

AME 3353
Design of Mechanical Components
Professor Harold L. Stalford

4.1 Fatigue Dynamics Loading Conditions

As found in the previous sections, some pre-load torques are present in the lug studs of cars. As explored in section 3.2, when the car is going straight, no forces act on the lug studs because there is no moment arm. However, extra stresses arise on these lug studs when the car turns and then it goes back to the pre-load value. Therefore, in this chapter, we are going to analyse the fatigue loading conditions, statics and stresses of the lug studs when the car goes through a turn. This situation is shown in figure 4.1-1 below.

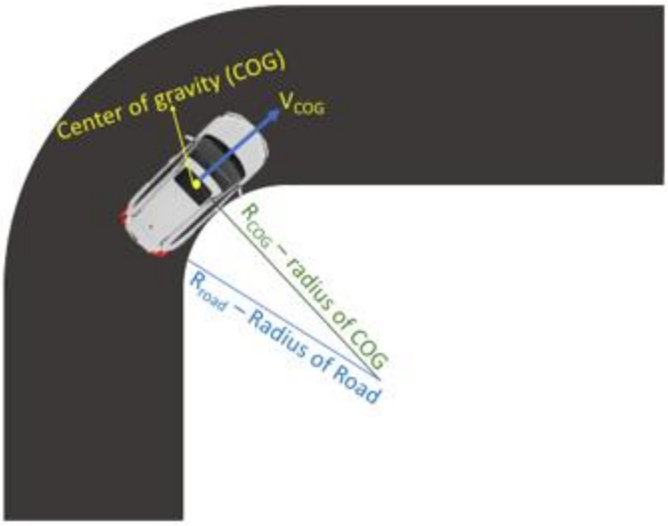

Figure 4.1-1: Toyota RAV4 going through a 90° turn

From the figure above, we will assume that the car is going on a road of 20ft or 6.096m.

through a turn of 20 ft or 6.7056m, which is R_{road}. We are going to assume that the center of gravity of the car is in the center of car, i.e. half of length and half of breadth – shown by the yellow dot in the middle of the car (COG). Therefore, by adding the radius of the road and half the width of the car will provide us with a center of rotation radius of the center of gravity of the car. This is shown in equation 4.1-1[1].

$$R_{COG} = R_{road} + \frac{width_{car}}{2} = 6.096m + \frac{1.84404m}{2} = 7.01802m \qquad (4.1\text{-}1)$$

Therefore, the radius of the center of gravity is 7.02m (equation 4.1-1). Using this, we can calculate the force exerted on the COG due to the circular motion. This can be calculated by equation 4.1-2.

$$F_{circ} = \frac{mv^2}{R_{COG}} \qquad (4.1\text{-}2)$$

We can plug in the respective mass of a loaded car (m), velocity of the car (V) and radius (R_{COG}) in equation 4.1-2.

$$F_{circ} = \frac{1982kg * \left(6.7056\frac{m}{s}\right)^2}{7.01802s} = 12700N \qquad (4.1\text{-}3)$$

For the velocity of 15 miles per hour or 6.7056 m/sec, we get 12700N of force on the center of gravity. If a sharper or tighter turn is taken, the radius becomes smaller and therefore, the centripetal force will be higher. This is how the force will change on the lug studs.

Since there are four tyres, we will assume that this force is spread evenly to all the tyres. Therefore, the force experienced by all the tyres is equation 4.1-4.

$$F_{tyre} = \frac{F_{circ}}{4} = \frac{12700}{4} = 3175N \qquad (4.1\text{-}4)$$

Even though the force is exerted on the tyres, generally the car does not slide on the road. The reaction force preventing the outward motion of the car is the friction force generated between the road and the tyres. The two forces – centripetal and friction force acting are equal to each other since there are no other forces acting in this direction. Therefore, now we can analyse the effect of this force on the mechanical component lug stud. The figure 4.1-2 shows the free body diagram of the right front wheel of the car.

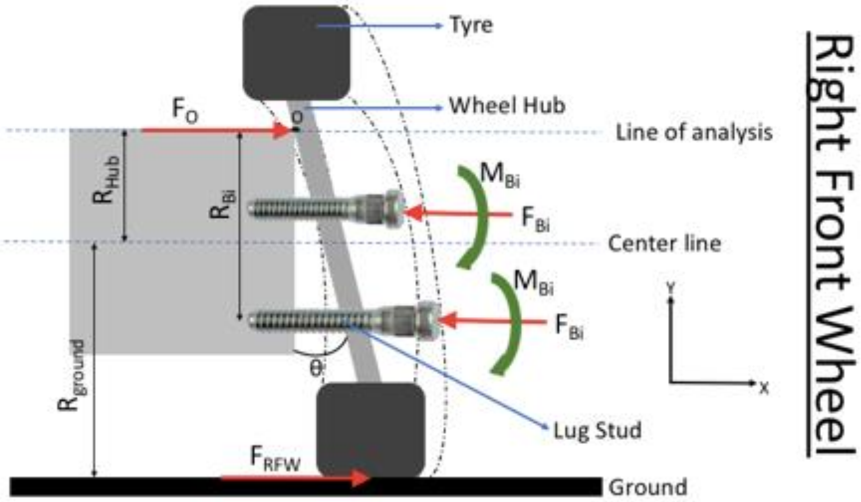

Figure 4.1-2: Free body diagram of the right front wheel

The hub of the Toyota RAV4 XLE has in total 5 lug studs arranged in a pentagonal fashion below in figure 4.1-3.

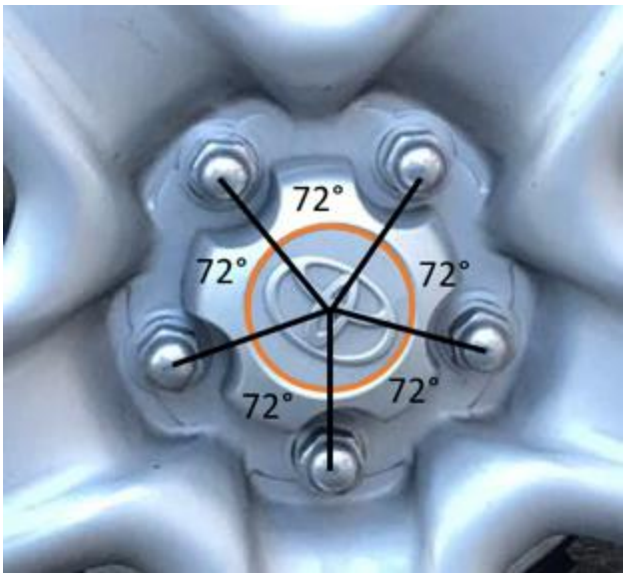

Figure 4.1-3: Arrangement of lug studs in a pentagonal fashion

Angle between every lug stud is 72°. We will calculate the force that acts on every lug stud based on these angles. We will start from the bottom stud being 0° followed by 72°, 144°, 216° and 288° going anti-clockwise.

From the free body diagram in the figure 4.1-2, we can calculate the equations of motion. First, we can find the balance of forces in the z direction.

$$\sum M_z = 0 \tag{4.1-5}$$

$$F_{RFW}(R_{Ground} + R_{Hub}) - \sum_{i=1}^{5} F_{Bi}R_{Bi} - \sum_{i=1}^{5} M_{Bi} = 0 \quad (4.1\text{-}6)$$

Similarly, we can also state the equations of motions for the forces in the x-direction. Since there are no forces acting in the y direction, we do not need to write down the equations for that.

$$\sum F_x = 0 \quad (4.1\text{-}7)$$

$$F_{RFW} + F_O - \sum_{i=1}^{5} F_{Bi} = 0 \quad (4.1\text{-}8)$$

The angle (θ) is unknown from figure 4.1-2. To find that, we can draw the following figure 4.1-4.

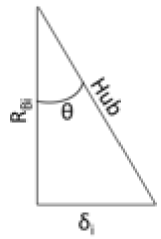

Figure 4.1-4: Triangle to find θ

Therefore, we can say that:

$$\tan(\theta) = \frac{\delta_i}{R_{Bi}} \quad (4.1\text{-}9)$$

Since this angle between the hub and the back of the wheel is really small, we can approximate it as:

$$\tan(\theta) \approx \theta = \frac{\delta_i}{R_{Bi}} \quad (4.1\text{-}10)$$

This equation can be rewritten as:

$$\delta_i = R_{Bi}\theta \quad (4.1\text{-}11)$$

Using the Hooke's law, we also have the relationship of the stress and the strain[2]:

$$\sigma_i = E\epsilon_i \quad (4.1\text{-}12)$$

From the definition of strain, we also have that:

$$\epsilon_i = \frac{\delta_i}{L} \quad (4.1\text{-}13)$$

Finally, we also know that the stress in equation 4.1-11 can be calculated as:

$$\sigma_i = \frac{F_{Bi}}{A_m} \quad (4.1\text{-}14)$$

Plugging equation 4.1-13 and 14 into 4.1-12, we get:

$$\frac{F_{Bi}}{A_m} = E\frac{\delta_i}{L} \quad (4.1\text{-}15)$$

We can replace the value of δ_i with equation 4.10-11. Therefore, the final equation we get for F_{Bi} is:

$$F_{Bi} = A_m E\left(\frac{R_{Bi}\theta}{L}\right) \quad (4.1\text{-}16)$$

From the solid mechanics, we are also aware that Moment can be written as:

$$\frac{M}{EI} = \frac{d\theta}{dx} \quad (4.1\text{-}17)$$

Since the car is turning in a circle, the angle of the hub does not change, therefore the right side of equation 4.1-17 is a constant. This can be rewritten as:

$$\frac{M_{Bi}}{EI} = \frac{\theta}{L} \quad (4.1\text{-}18)$$

Rearranging the equation to find the moment:

$$M_{Bi} = \frac{EI\theta}{L} \tag{4.1-19}$$

In the moment, all the parameters above inertia (I), Young's modulus (E), angle (θ) and the length (L) stays constant. Therefore, the moment on each of the lug stud remains as constant. Equation 4.1-17 and 4.1-19 can be plugged into the equation 4.1-6 to find:

$$F_{RFW}(R_{Ground} + R_{Hub}) = \sum_{i=1}^{5} A_m E\left(\frac{R_{Bi}^2\theta}{L}\right) + \sum_{i=1}^{5}\frac{EI\theta}{L} \tag{4.1-20}$$

Since $\frac{\theta}{L}$ appears in both the summations and is a constant, it can be taken out. A_m,E, I and L are constants, so they can be taken out of the summations also. This changed equation looks like:

$$F_{RFW}(R_{Ground} + R_{Hub}) = \frac{\theta}{L}\left[A_m E\sum_{i=1}^{5}\left(R_{Bi}^2\right) + NEI\right] \tag{4.1-21}$$

Equation 4.1-21 can be rearranged to solve for $\frac{\theta}{L}$ as this is the only unknown in the equation. Everything else is either measured or is known.

$$\frac{\theta}{L} = \frac{F_{RFW}(R_{Ground} + R_{Hub})}{\left[A_m E\sum_{i=1}^{5}\left(R_{Bi}^2\right) + NEI\right]} \tag{4.1-22}$$

Equation 4.1-22 can be plugged into equation 4.1-16 to find the value of the forces in each of the lug stud.

$$F_{Bi} = A_m E R_{Bi} * \frac{F_{RFW}(R_{Ground} + R_{Hub})}{\left[A_m E\sum_{i=1}^{5}\left(R_{Bi}^2\right) + NEI\right]} \tag{4.1-23}$$

Simplifying the equation 4.1-23 gives:

$$F_{Bi} = F_{RFW} * \frac{R_{Bi}(R_{Ground} + R_{Hub})}{\left[\sum_{i=1}^{5}\left(R_{Bi}^2\right) + \frac{NI}{A_m}\right]} \tag{4.1-24}$$

In equation 4.1-24, all the parameters are known to find the forces in each of the lug stud. R_{Bi} is not measured but it can be calculated as it just the distance from the top (point) to the lug stud. A simple formula can locate the distance based on the angle from the bottom stud going counter-clockwise. This formula is:

$$R_{Bi} = R_{hub} + R_{config}\cos(\theta) \tag{4.1-25}$$

All the 5 lengths can be calculated like this. We will assume that 1 lug stud is perfectly in the middle at the bottom as shown in figure 4.1-3. F_{RFW} can be found using the equation 4.1-2 and dividing it by 4 to find for the individual tyre. This value is assumed to be a constant of 3175N. N is the number of bolts, which is 5. We can also assume that the lug stud is a cylinder and therefore, calculate the moment of inertia using the equation:

$$I_x = \frac{1}{4}\pi r^4 \tag{12}$$

Therefore, we can calculate the force on the bottom (first – 0°) with a speed of 15 mph or 6.7056 $\frac{m}{s}$ by combining equation 4.1-2, 4.1-25, 4.1-26 and 4.1-24.

$$F_{Bi} = 3175N * \frac{0.14m(0.162m + .08m)}{\left((.14^2 + .099^2 + .031^2 + .31^2 + .099^2)m^2 + \frac{5 * \frac{1}{4}\pi\left(\frac{.012}{2}\right)^4 m^4}{\pi.\left(\frac{012}{2}\right)^2 m^2}\right)} = 2621N \tag{4.1-27}$$

Since the wheel goes into rotation, all of the studs go through the force exerted above as it rotates 360° while turning.

The table 4.1-1 below shows the maximum force for the speeds of 15, 20, 25 and 30 mph.

Table 4.1-1: Speeds and respective forces on the lug stud

Speed (mph)	Speed (m/s)	Speed (in/sec)	F_{RFW} (N)	F_{RFW} (lbf) $\frac{lbm*in}{sec^2}$
15	6.7056	264	3175	275580
20	8.9408	352	4659	404398
25	11.176	440	7280	631872
30	13.4112	528	10483	909896

In a similar fashion, the moment for these forces can also be calculated. However, as mentioned earlier the moment is constant for all the bolts for constant speed. We can plug the value we for $\frac{\theta}{L}$ from equation 4.1-22 into equation 4.1-19.

$$M_{Bi} = EI\frac{F_{RFW}(R_{Ground} + R_{Hub})}{\left[A_m E \sum_{i=1}^{5}(R_{Bi}^2) + NEI\right]} \quad (4.1\text{-}28)$$

This can be further simplified as;

$$M_{Bi} = \frac{F_{RFW}(R_{Ground} + R_{Hub})}{\left[\frac{A_m}{I}\sum_{i=1}^{5}(R_{Bi}^2) + N\right]} \quad (4.1\text{-}29)$$

Therefore, the moment for various speeds can be found like this. The moment for 15 mph or 6.7056 m/s is:

$$M_{Bi} = \frac{3175m(0.162m + 0.08m)}{\left[\frac{\pi\left(\frac{.012}{2}\right)^2}{\frac{\pi}{4}\left(\frac{.012}{2}\right)^4}\sum_{i=1}^{5}(R_{Bi}^2) + 5\right]} \quad (4.1\text{-}30)$$

The table 4.1-2 lists all the moment values for various speeds.

Table 4.1-2: Moments for the corresponding velocities on the lug stud

Speed (mph)	Speed (m/s)	Speed (in/sec)	M_{Bi} (Nm)	M_{Bi} (lbf in) $\frac{lbm*in^2}{sec^2}$
15	6.7056	264	3175	275580
20	8.9408	352	4659	404398
25	11.176	440	7280	631872
30	13.4112	528	10483	909896

In this section, we have found the maximum force and moment on the lug stud when a car turns 90° in a 20 ft radius or 6.096m road.

4.2 Fatigue Stresses

Similar to the previous sections, the fatigue stresses can be calculated on the lug stud. Two kinds of stresses will be acting – axial and bending. There are no shear stresses acting as mentioned in chapter 3 that there is no moment arm when the force acts from the top on the hub.

The maximum axial stress for each lug stud is calculated using the formula:

$$\sigma_i = \frac{F_{Bi}}{A_m} \quad (4.1\text{-}31)$$

Since the maximum forces are already known from table 4.1-1 and A_m has been calculated from chapter 2, the stresses can be calculated easily. These are listed below in table 4.2-1.

Table 4.2-1: Axial stresses for all the velocities on lug stud

Speed (mph)	Speed (m/s)	Speed (in/sec)	σ (MPa)	σ (psi) $\frac{lbm}{in*sec^2}$
15	6.7056	264	23.17	1297617
20	8.9408	352	41.20	2306875
25	11.176	440	64.37	3604492
30	13.4112	528	92.69	5190469

Similarly, the moment in a cantilever beam can be calculated as:

$$\sigma_i = \frac{yM_{Bi}}{I} \quad (4.1\text{-}32)$$

y in this case is $\frac{dm}{2}$. So plugging it into the equation 4.1-32 gives:

$$\sigma_i = \frac{dm * M_{Bi}}{2I} \quad (4.1\text{-}33)$$

Using this, we can calculate the bending stress on all the lug studs at the bottom when the RAV4 goes around a corner of radius 20ft or 6.096m.

Table 4.2-2: Bending stresses for all the velocities on lug stud

Speed (mph)	Speed (m/s)	Speed (in/sec)	σ (MPa)	σ (psi) $\frac{lbm}{in*sec^2}$
15	6.7056	264	0.99	55612
20	8.9408	352	1.77	98866
25	11.176	440	2.78	154478
30	13.4112	528	3.97	222489

Since the bending stress is acting in the clockwise direction as shown in the figure 4.2-1, the total stress at the bottom of the lug stud ends up being the maximum.

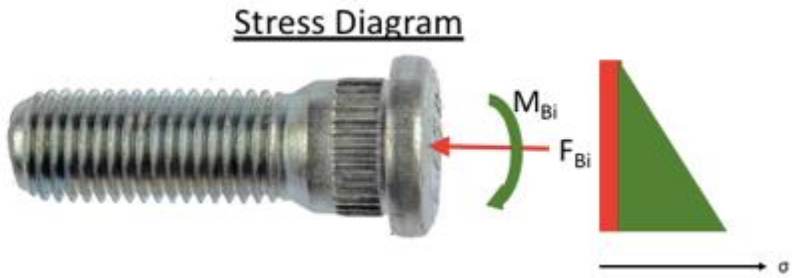

Figure 5: Stress diagram of lug stud due to axial force and bending moment

Therefore, the bottom most part of the stud experiences the most amount of stresses.

Both, the bending and the axial stresses can be added up to find the maximum and have been tabulated in the table 4.2-3 below.

Table 4.2-3: Total stresses of the lug stud

Speed (mph)	Speed (m/s)	Speed (in/sec)	σ_{total} (MPa)	σ (psi) $\frac{lbm}{in*sec^2}$
15	6.7056	264	24.16	1353229
20	8.9408	352	42.97	2405741
25	11.176	440	67.15	3758970
30	13.4112	528	96.66	5412958

All the total stresses have been tabulated in the table 4.2-3 above.

4.3 Fatigue Operating Point Plot

At first, we have calculate the half of the total stresses calculated above. This is to find the mean of the stresses (the stress at the middle of the stud). These mean values have been given below.

Table 4.3-1: sig_m values

Speed (mph)	Speed (m/s)	Speed (in/sec)	σ_m (MPa)	σ_m (psi) $\frac{lbm}{in*sec^2}$
15	6.7056	264	12.08	676614.5
20	8.9408	352	21.49	1202871
25	11.176	440	33.58	1879485
30	13.4112	528	48.33	2706479

On these σ_m, if we add the preload stresses, then we can find the maximum stress on the lug stud at any point.

Table 4.3-2: sig_a values

Speed (mph)	Speed (m/s)	Speed (in/sec)	σ_a (MPa)	σ_a (psi) $\frac{lbm}{in*sec^2}$
15	6.7056	264	89.98	687961.5
20	8.9408	352	99.385	1214217.5
25	11.176	440	111.475	1890832
30	13.4112	528	126.23	2717826

Now that we know the maximum stresses, we can plot them on a graph. The table for the graph is given below:

Table 4.3-3: Table of σ_m and σ_a

σ_a	σ_m
89.98	12.08
99.385	21.485
111.475	33.575
126.23	48.33

Figure 4.3-1: Graph of σ_m vs. σ_a

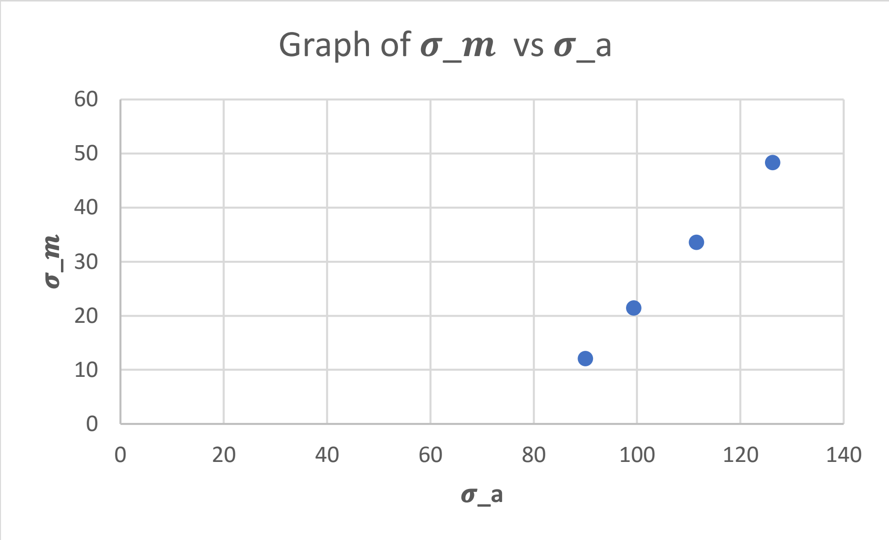

Table 5: All the parameters used in fatigue analysis

Serial number	Parameter
1	dm
2	Preload Torque T
3	Eq. (10.4) calculation of Tensile Force in Lug stud/bolt Tensile force: W=
4	Am
5	Using Tensile Force W from Eq. (10.4) Tensile stress due to preload torque: σ-preload =W/ Am
6	Yield stress Sy
7	Using Tensile Force W from Eq. (10.4) Yield Safety Factor: Sy/σ-preload
8	Speed V
9	Max mass Vehicle (kg); Max weight Vehicle (lbs)
10	Radius of turn for right front wheel
11	Half width of vehicle
12	R-cg Radius of turn for vehicle cg (center of mass)
13	Fc: Centripetal force acting at cg
14	F-RFW: Force on front wheel where tire meets road
15	R-hub
16	R-gnd (i.e., R-ground)
17	R-hub + R-gnd
18	R-rhc (rim holes conf.)
19	R-B1 distance from pivot to bolt1: use φ1=0 R-B1=R-hub + R-rhc*cos(φ1)

20	R-B2 distance from pivot to bolt2
21	R-B3 distance from pivot to bolt3
22	R-B4 distance from pivot to bolt4
23	R-B5 distance from pivot to bolt5
24	R-B6 distance from pivot to bolt6
25	Sum of all (R-Bi)^2 from i=1,…,N (N bolts)
26	I moment of inertia for bolt cross-section with dm
27	N (I/Am)
28	[Sum(R-Bi)^2 + N (I/Am)]
29	X1=RB1*(Rgnd + Rhub)]/[Sum(R-Bi)^2 + N (I/Am)]
30	F1=X1* F-RFW
31	σ Axial-B1= F1/ Am [additional tensile stress on bolt 1 due to axial force]
32	I/[(RB1^2)*Am]
33	Y1= X1*[I /(RB1^2)*Am]
34	M1= Y1*RB1* F-RFW
35	σ Bending-B1=[dm/2]*M1/ I [additional tensile stress on bolt 1 due to bending]
36	σ Axial-B1 + σ Bending-B1 [additional tensile stress on bolt 1 due to turn]
37	σ-a = [σ Axial-B1 + σ Bending-B1]/2
38	σ-m = σ-preload + σ-a

The car goes through numerous cycles throughout the year. We assume that that every day car takes around 100 turns. Based on an average life of 15 years, this is 547500 cycles. This is a lot of cycles and in the next section, we will calculate the life expectancy of a lug stud.

4.4 Table of Data and Results

Table 4.4-1 Table of all the results

Speed (mph)	**Speed (m/s)**	**Speed (in/sec)**	F_{RFW} **(N)**	F_{RFW} **(lbf)** $\frac{lbm*in}{sec^2}$
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Speed (mph)	Speed (m/s)	Speed (in/sec)	σ_m (MPa)	σ_m (psi) $\frac{lbm}{in*sec^2}$
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σ_a	σ_m
89.98	12.08
99.385	21.485
111.475	33.575
126.23	48.33

4.5 References

[1] "Coriolis Force." *Centripetal Force*, hyperphysics.phy-astr.gsu.edu/hbase/corf.html.

[2] hyperphysics.phy-astr.gsu.edu/hbase/permot2.html.

4.6 Level of Effort

I spent about 20 hours on this report. I found the lecture to be very exciting full of equations. Thank you.